Chapter 1

MULTIMEDIA SIGNAL PROCESSING

Introduction

Introduction

- Digital image & video processing : a new domain and technology
  - Mathematics of information
  - Signal processing
  - Electronic & optronics systems
  - Computer sciences and micro-processors (VLSI, DSP)
- Lots of methods
  - ad'hoc & founded as well
  - Needs of methodology
- Widespectrum of applications

When we speak of *digital image processing*, we mean the set of techniques used to modify a digital image in order to improve it (in terms of quality), or to reduce its size (in terms of bits compression encoding) or to get information out of it. Processing digital images is a new sector of knowledge, which has quickly developed thanks to the emergence of new information technologies. It relies mainly on the mathematics linked to information, signal processing, electronic systems and the advance in microprocessor computation capacities, particularly those that have been exclusively developed for signal processing and which offer high computation speed and capacity (DSP, etc.).

As digital image processing is in its early stages and its scope of application is quite spread, it has quickly become apparent that a methodology must be created and the domains of application separated out. Image processing can now be seen as four distinct fields of action: analysis, synthesis, coding and quality enhancement.

To start with, a description of these four domains will allow us to better understand what image processing really is. Following this, we will look at how to obtain the digital images (by digitization of analog images) that we wish to process.

Introduction

A picture is worth one thousand words
and a video is worth one thousand sentences

Rich info. from visual data.

Examples of images around us

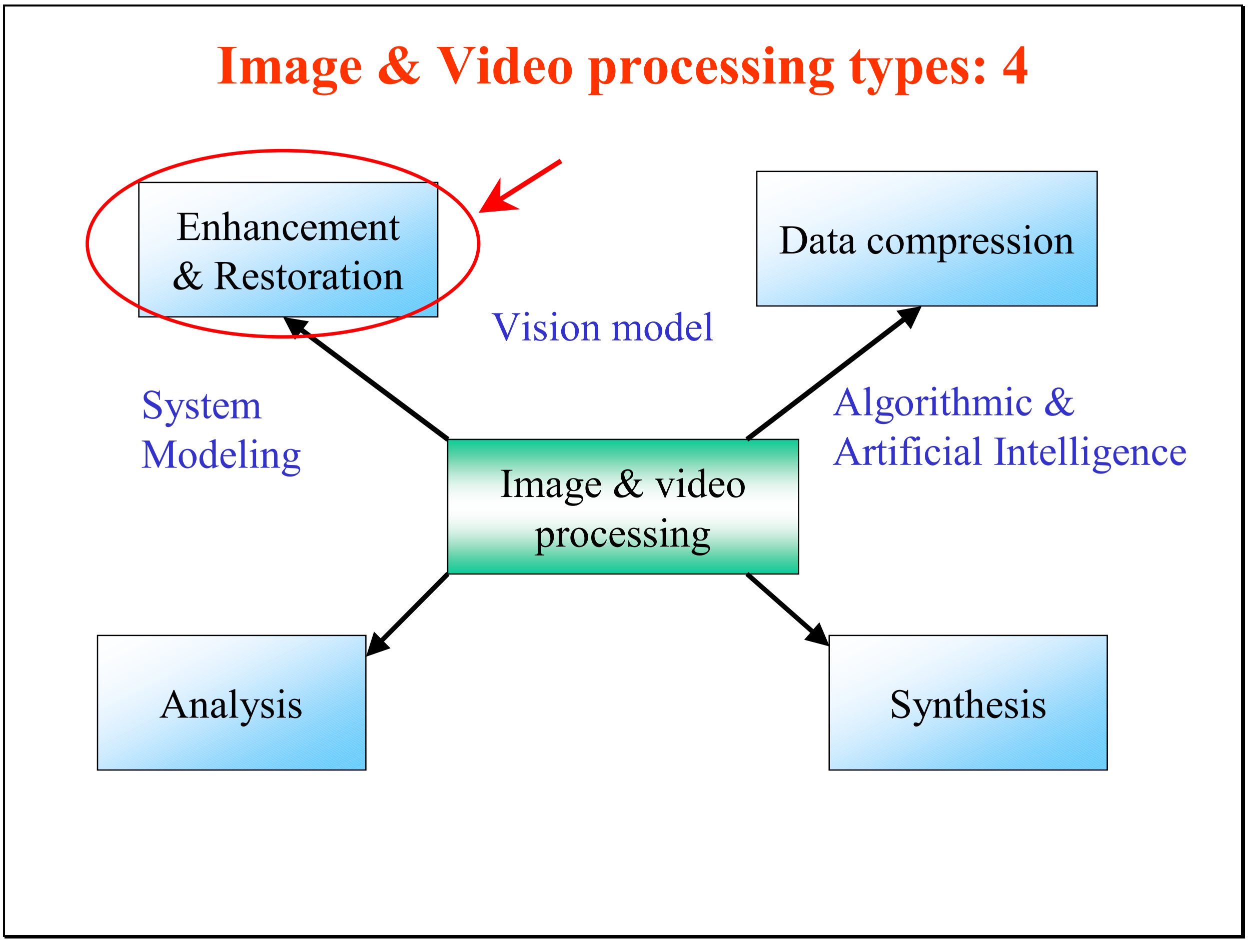
- natural photographic images
- artistic and engineering drawings
- scientific images (satellite, medical, etc.)

"Motion pictures" => video

- movie, TV program, news
- family video
- surveillance and highway/ferry camera

We come in contact with all sorts of images in our daily environment: photographs of landscapes, of people, computer-generated drawings and paintings, images from medical radiology, satellite images, and so on. Some of these images (such as via satellites and for medicine) cannot be directly observed, while others have characteristics that can be extracted automatically, stored and sent. The processing that could be carried out on these images is highly varied, as the images we meet in our environment are also varied, by their nature and properties as well as the scenes they describe. They are all different but, evidently, it is not conceivable to create a specific type of processing for each one. This has led to a classification of image processing that does not rely on image characteristics, but rather on the objective of the processing. We can distinguish four types of domains of application for the digital image processing:

- restoration and enhancement image,
- image analysis,
- image coding with data compression,
- image synthesis.



Let's now look in detail at the four fields of action linked to digital image processing.

Enhancement and restoration:

Let's consider an observed image I_0 with which we associate a signal s_0 that we model as: $\mathbf{s_0 = \mathit{f}(s_u, d, b)}$

where: - s_u is the usable image signal obtained from an ideal image (without any loss);
- d is the distortion function which operates on the ideal image (geometric distortions, blur, etc.);
- b is noise;
- f is an observation function dependant of these two signals and of the distortion function.

The processing carried out on I_0, which will output the transformed image I_T, must enable the information contained in I_T to be used in a more efficient manner than the information in the directly observed image I_0. If a change occurs in the presentation characteristics of the image, we can talk of an *enhancement process*, whereas if there is a partial inversion in the quality loss, we can talk of a *restoration process* (for example, 2D linear filters, 2D adaptive filters etc.).

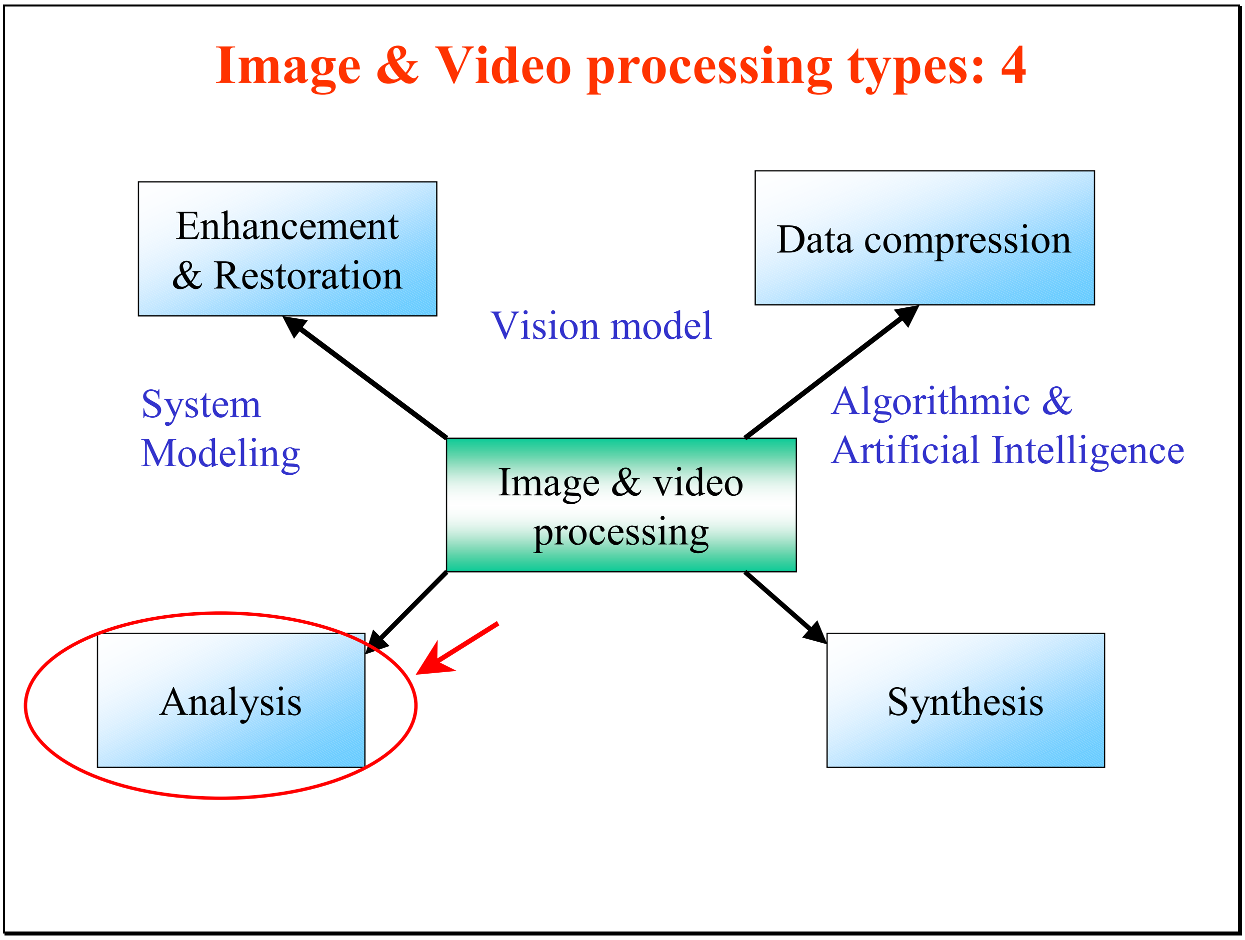


Image analysis:

This refers to partially or fully describing the scene from the observed image (objects detection, object dimensions, position in space, etc.).
Classically, this analysis process takes place in 3 successive stages:

- pre-processing and extraction of the characteristic traits,
- classification and pattern recognition,
- description and possibly interpretation of the image content.

Note however that image analysis also varies depending on the support medium: the problems raised by the analysis of 2D images, of 3D images or of moving images are quite numerous, with a special attention to moving images due to the techniques used in that case and the nature of the required objectives.

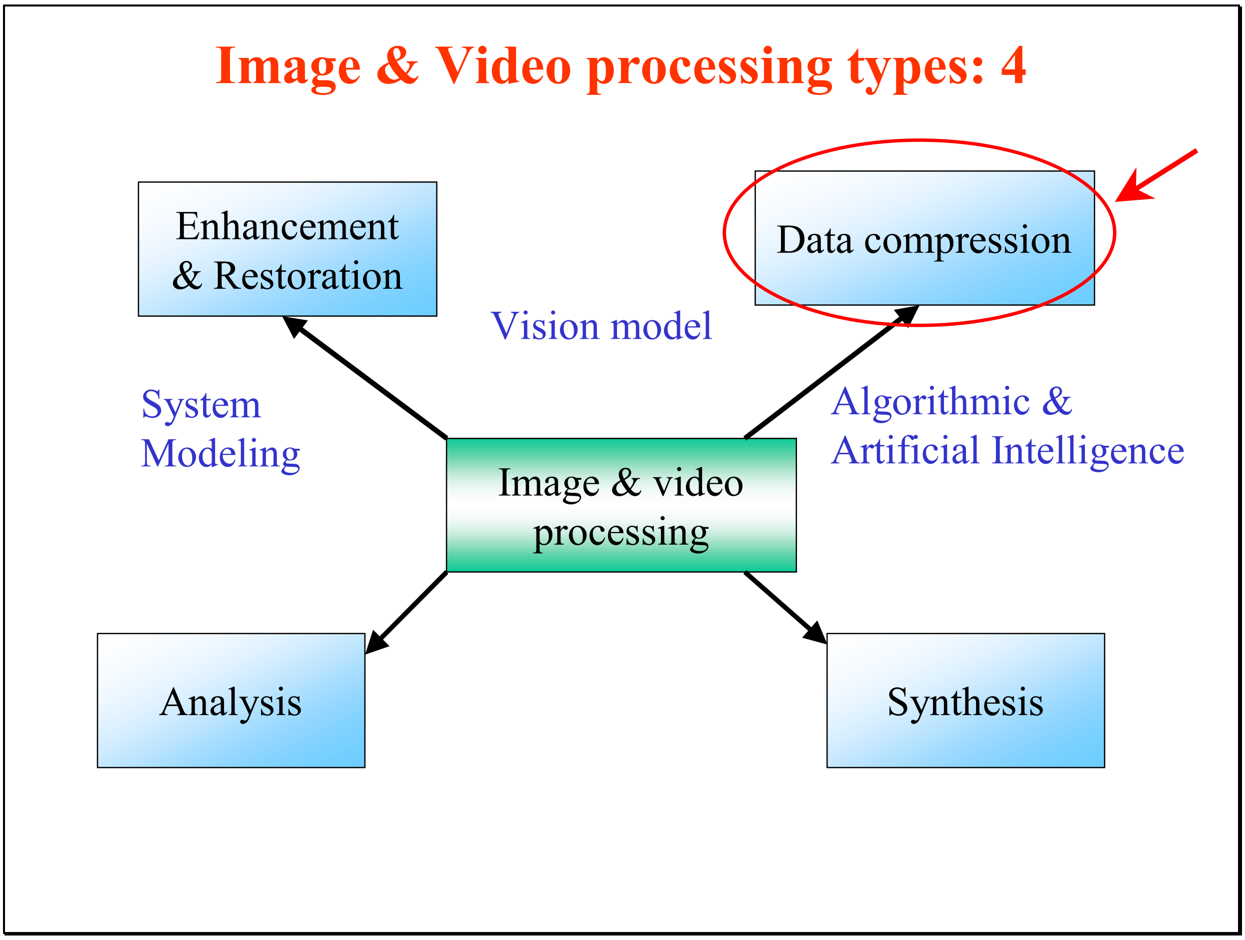


Image coding with data compression:

The basis for representing a digital image is a rectangular 2D table of elements called pixels. This implies to handle and memorize a large amount of pixels. For a simple grayscale image, typically 512×512 pixels (256 000 pixels) and 8 bits per pixel must be memorized to obtain a good resolution (8 bits to code one pixel, which gives per pixel a number of $2^8 = 256$ possible values). For high-resolution color images, or for moving images, the number of bits necessary for image representation quickly becomes enormous to store or send it.
Encoding an image aims to obtain a representation of that image that requires a greatly reduced number of bits in comparison with the original image representation.
To measure this reduction, we use a compression rate τ defined as:

$$\tau = \frac{\text{number of bits representing the basic image representation}}{\text{number of bits representing the image after encoding}}$$

This rate must be higher than 1 for real compression to take place.

As the number of bits for the basic image representation is fixed, the compression rate is in fact inversely proportional to the number of bits representing the image after encoding. If you want to make an exact reconstruction of the image after decoding, you need to use reversible encoding. This creates a constraint that means that the compression rate is often rather low. To increase the compression rate significantly, you need only to rely on a representation that is visually exact. In this case, the human eye will perceive no difference between the original image and the image that is reconstituted after decoding. In addition, the complexity of the encoding/decoding must be limited. Encoding a digital image involves finding a healthy balance between the compression rate that will be high enough to make data storage and transmission easier but that will not unduly affect the picture quality, and simultaneously keeping in mind that decoding complexity must be restrained.

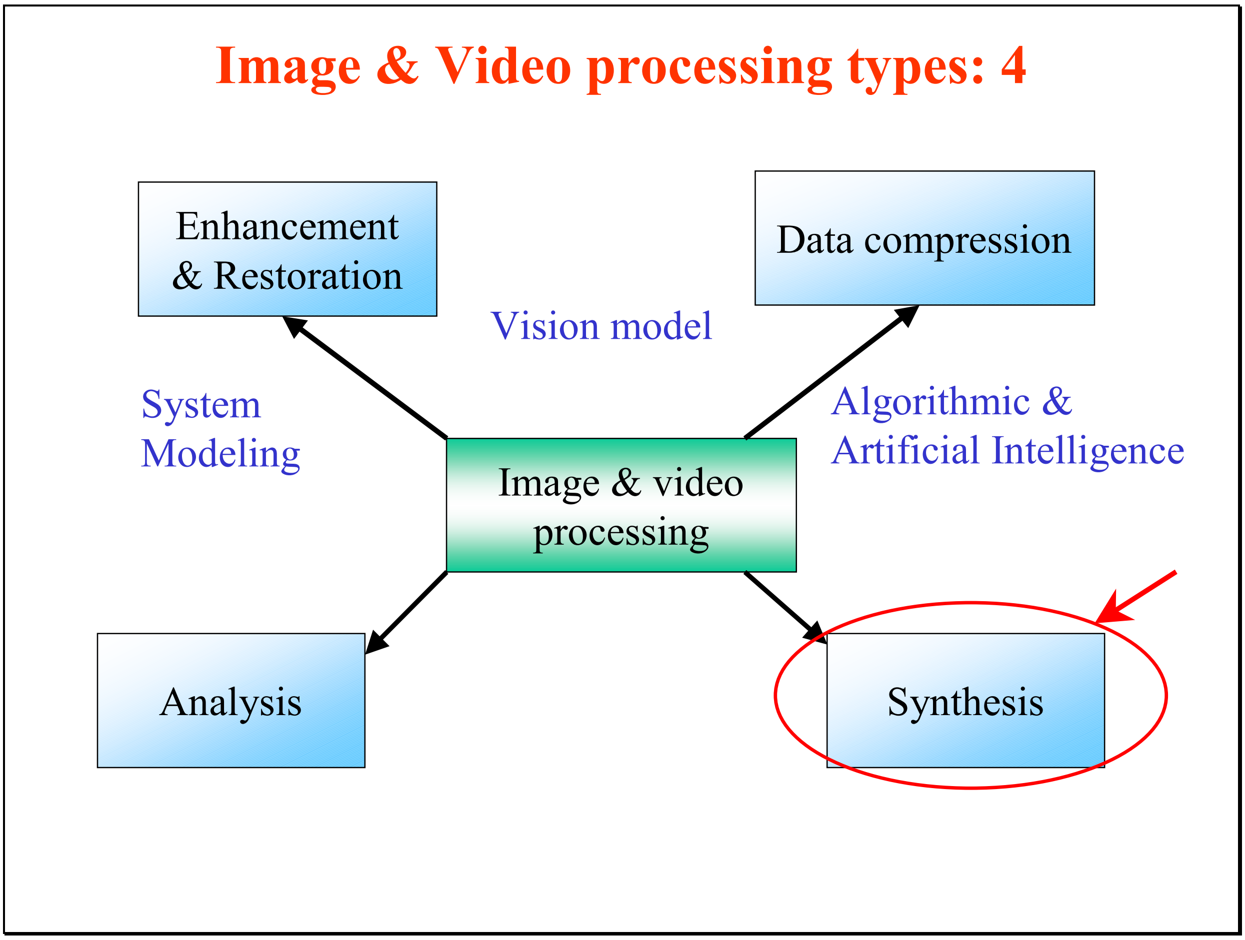
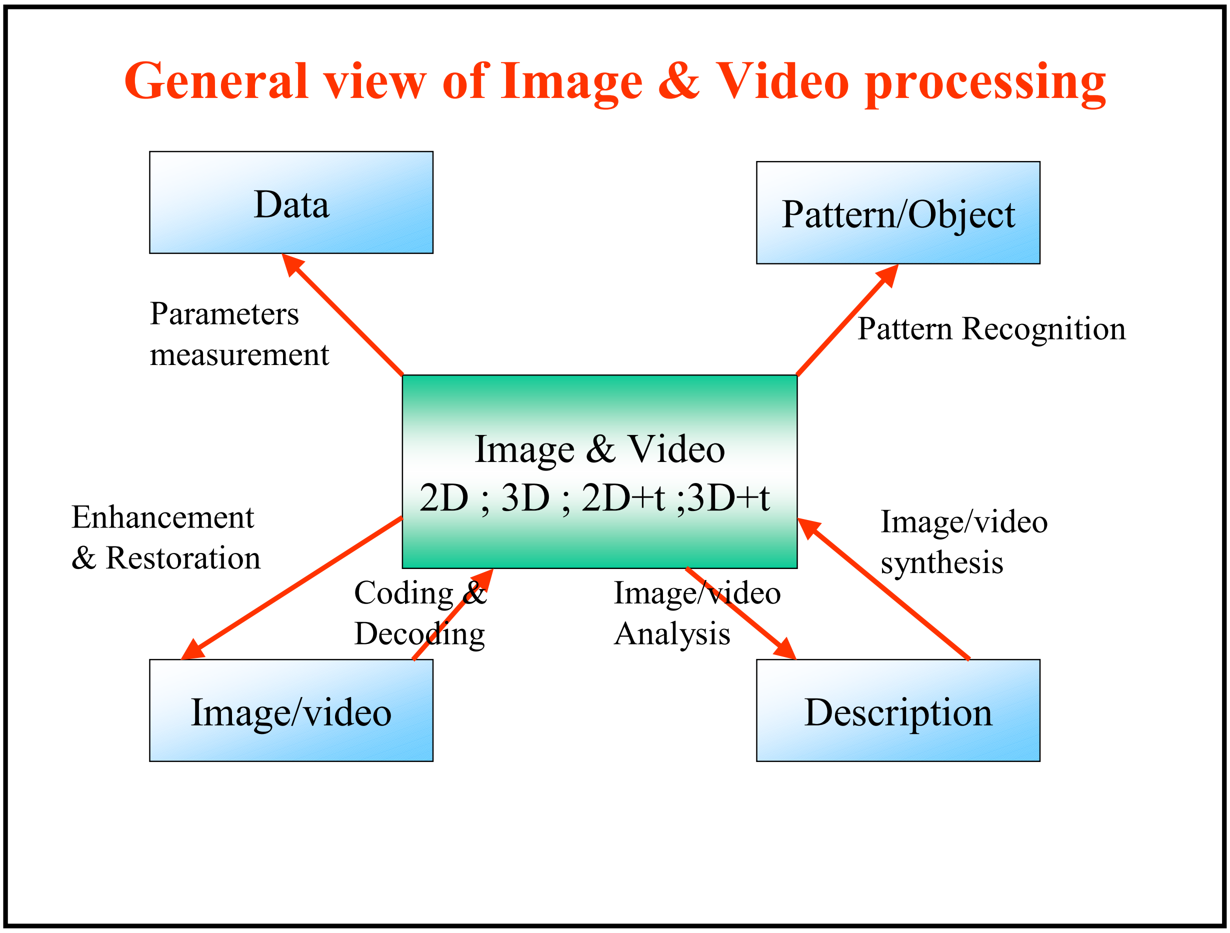


Image synthesis:

The goal of this is to reconstruct an image that resembles an image from the simulated scene, from a description of the scene, the objects making it up, its lighting characteristics (e.g. lightening orientation, intensity) as well as the capture device (CCD or CMOS camera, etc.). This reconstructed scene may resemble reality or be purely fictional. The first applications concerning image synthesis were oriented towards training simulators (flight and vehicle simulators) before involving out into other domains (audiovisual, cinema, art etc.).

We should point out that image processing also relies on studies linked to **the structure of processing machines**. An image contains a significant amount of data. In fact, for a moving image there are $N \times M \times P$ samples per second to process (N dots per line, M lines and P images per second). Image processing requires powerful calculation capacity. It needs high performance architectures with high degrees of parallelism and significant processing speeds.

Digital image processing has just been presented in accordance with the four main domains of application. We can now look at this from another angle, by concentrating of the nature of the processing results.



We can characterize image processing not simply in terms of its domains of application, but also according to the nature of the results that will be put out. There can be two types of input: an image or a description.

- From an image input

The output may be:

- image type
  This is the case with image coding for data compression, image enhancement and restoration of poor quality images; these three have already been presented.

- data type
  This is the case when you make an elementary image analysis. You are interested in the spatial dimensions of an object in the scene, its position or its color.

- pattern type
  This is also a case of image analysis, but a more elaborate one. This involves extracting and recognizing the objects observed in the scene.

- scene description type

  This is also a possible output for an image analysis, but in the most advanced version. The image is entirely broken up so that each object present in the scene can be recognized. The scene is described in its totality and can be interpreted.

▪ From a description input

For the output, the only expected type is an image. The domain of application involved is *image synthesis*. We wish to reconstruct the image according to a given description. What objects are present? What are their dimensions? Where are they in the scene? How are they lit? What are the parameters (focal length, viewing angle etc.) of the camera doing the filming?

You have now studied image processing from two different aspects: these aspects are nevertheless strongly interconnected. We can now go on to look at the characteristics of the signal that we wish to process: the image.

Basic Image/Video Properties

- Image/video signal is a 2-D/2-D+t scalar/vectorial signal which is complex:
    - Non stationary
    - Non Gaussian
    - Non isotropic
- Two (*image*) or three (*video*) main features
    - Contours: abrupt change in some important characteristics
    - Texture: spatial variation of the 2-D signal apart its local mean value (mean value, texture description)

      Edges are locally 1-D signals
    - Motion: in case of video (time-varying image sequence)

An image is a 2D scalar (grayscale image) or vectorial (color image for example) signal. Video which is a succession of images ordered temporally is a 2-D+t (t: time) scalar (gray-scale video) or vectorial (color video) signal.
The "Image" signal is complex because it is:

- non-stationary: its contents in space frequencies change with the space coordinates;
- non-Gaussian: its statistical properties do not follow a Gaussian probability law;
- non-isotropic: the properties of the image signal are not the same ones with the orientation (e.g. in the images taken on the ground, the horizontal and vertical directions are more frequent for contours than the oblique directions).

The classical methods and tools used in signal processing are often designed for stationary, Gaussian or isotropic signals (e.g. Discrete Fourier Transform). They cannot be directly applied to images.

Incidentally, images are mainly characterized by two types of element, namely ***contour*** and ***texture***:

- Contours are abrupt change of important characteristics from an area A to an area B of a scene (average value, texture description). The edges can be locally considered as 1D signals;
- Textures are spatial variation of the 2D signal apart its local mean value;
- Motion of the objects in a scene involves temporal modifications in the successive frames of a video.

Now you have seen a general overview of images and digital image processing. In the next part of this chapter, we will look at methods for representing images (digitizing and encoding), which must be used in order to carry out digital processing. We will also see some concrete examples of results relating to the domains of application that we explored earlier.

Chapter 1

MULTIMEDIA SIGNAL PROCESSING

Examples of Image Processing

Images and Videos Analysis

The four major domains of application for image processing have been globally presented: what are the issues? what are the objectives? what methods to use? and which tools? We will now look more closely at some concrete applications derived from these different domains.

Image and video analysis

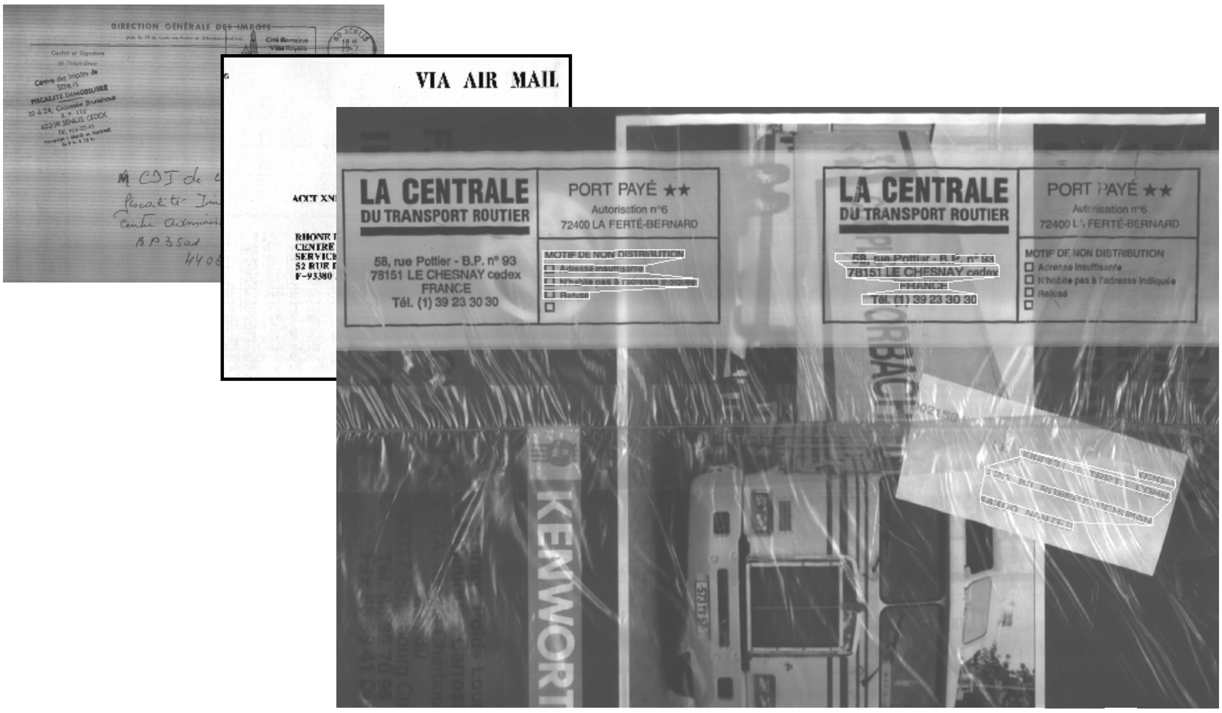
- **Objectives**
  - Objects detection and extraction (segmentation)
    Objects of interest
  - Pattern recognition
    Object classification, object identification
  - Scene analysis and interpretation
    - Relational
    - Quantitative description
    - Qualitative description

The first domain that interests us is image analysis. This covers a large number of potential applications and is probably the domain for which there is the greatest variety of examples (medical analysis, classification of chromosomes, automatic reading, signature verification, remote sensing, segmentation and classification of geographical areas, etc.).
We can however suggest a typical image analysis cycle, which consists of

- ***Detecting*** the different objects present in a scene and extracting them (segmentation, slicing into areas).
- ***Characterizing*** the objects in the picture (size, location, etc.).
- ***Recognizing*** objects and classifying them.
- ***Analyzing and interpreting*** the scene according to the previous results. The scene could for example be described quantitatively and/or qualitatively.

Let us remember though that the applications are quite varied and that the global approach given here may sometimes require changes in the stages of processing, according to the difficulties encountered.

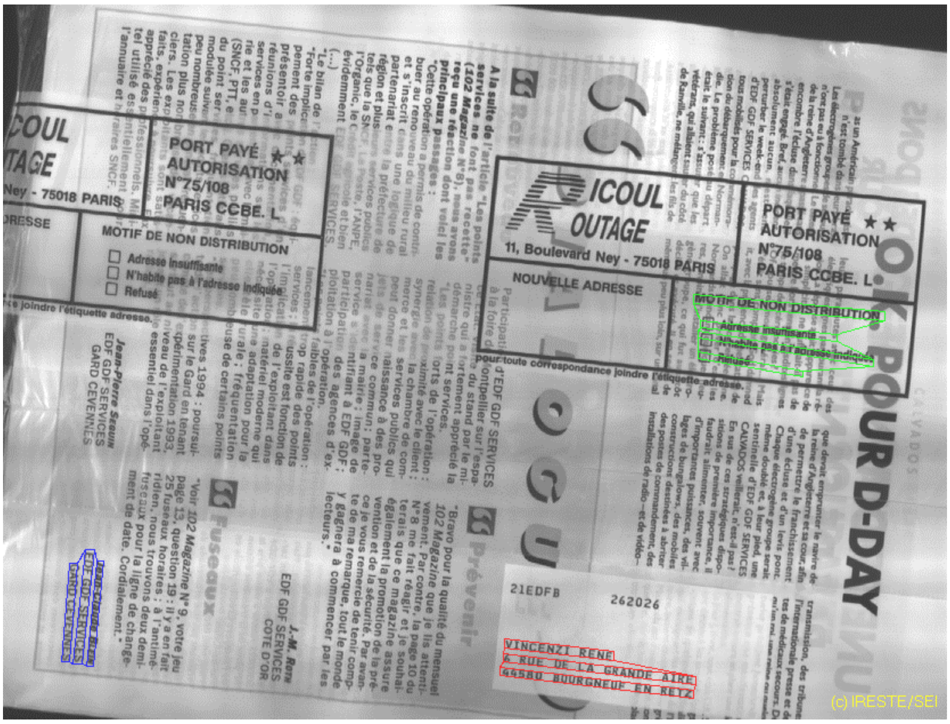
Address block detection and extraction



The example presented in the image above highlights detection and extraction of a delivery address. The application would be used for automatic sorting whose nature would depend on the postal objects and how they were posted. Most industrialized nations use automatic sorting and obstacles vary from one country to another due to differences that exist between postal codes (among other factors); are the postcodes numbers or other characters ? what is their size ? etc.

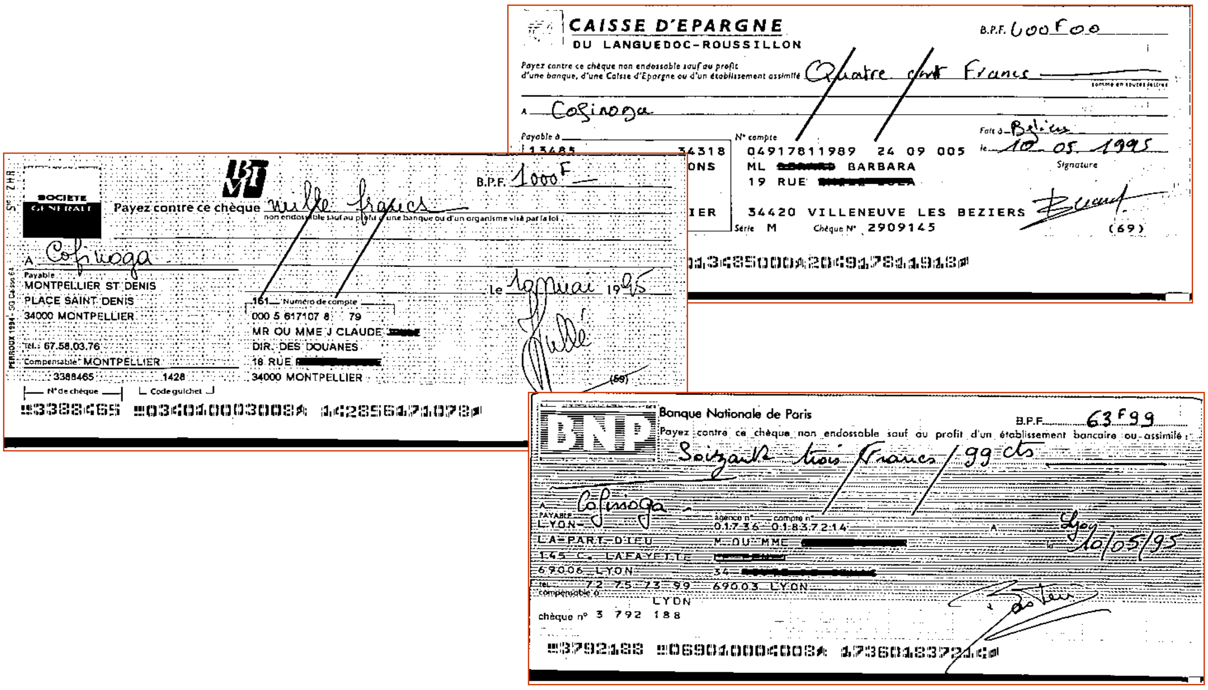
Various processing algorithms exist for this. We can for example notice that the characters of the text form a regular texture containing vertical contours arranged horizontally. By applying directional filters, it is possible to highlight the pixels of the image that are linked to text. We could for example carry out binarization followed by post-processing so we can extract the rectangles surrounding text boxes. Once the text is separated from the image, we could analyze the neighboring components to find those that correspond to text boxes. Once the global processing of the image was carried out, you would then need to apply local processes and consider various constraints (for example, geometrical) to avoid errors. These different operations are described in the next chapters.

Address block detection and extraction



The image above is also the result of an analyzing process. The objective is once again to detect and extract from the image the text boxes linked to the recipient’s address. However, the image above presents new difficulties; the background is much less even than the one given previously, as it is made up of text. We must also be able to distinguish, from all the overlapping characters, which ones are linked to the "sender’s address block" and which are linked to the rest of the image. However the background text and the address text are perpendicular to each other. It would seem possible to segment the image by applying adapted directional filters. Then we could take up the Characterization – Recognition – Interpretation chain from an image analysis process so we could obtain, as on the picture shown, exact detection of the recipient’s address block.

Extracting and reading the amount written on a cheque



The last example presented for image analysis concerns extracting and reading the amount written on a cheque. This is particularly important, considering the industrial and economic potential. Many publications and methods exist concerning this subject, however potential users of this kind of automatic recognition seem to want more reliable results. Because of this, the problem remains open-ended. With this technology, figures and handwritten letters need to be recognized in an environment whose complexity demands the use of appropriate techniques. Apart from the enormous variation in how individuals may form characters, other problems arise such as the non-standardized format of cheques (field position, type of background) and overlapping of the items that make up the amount (figures, baseline, decimal separator).

The main products or prototypes available in France come from SRTP (the post office) with Dassault AT (cheque processing automat that reads only the numerical amount), Matra (MSI) and their LireCheques (cheque reader) product and finally the product called InterCheque from a2ia that can be found in some banks (Société Générale, Banque Populaire) whose recognition rate approaches 70% thanks to a technology based on neural networks and Markov models.

Examples of Image Processing

Image and Video Coding with Data Compression

After these few examples of image analysis applications, we are going to look now at applications linked to image encoding for data compression.

Context

- Extension of digital techniques to many domains of information processing , information capture , storage and transmission

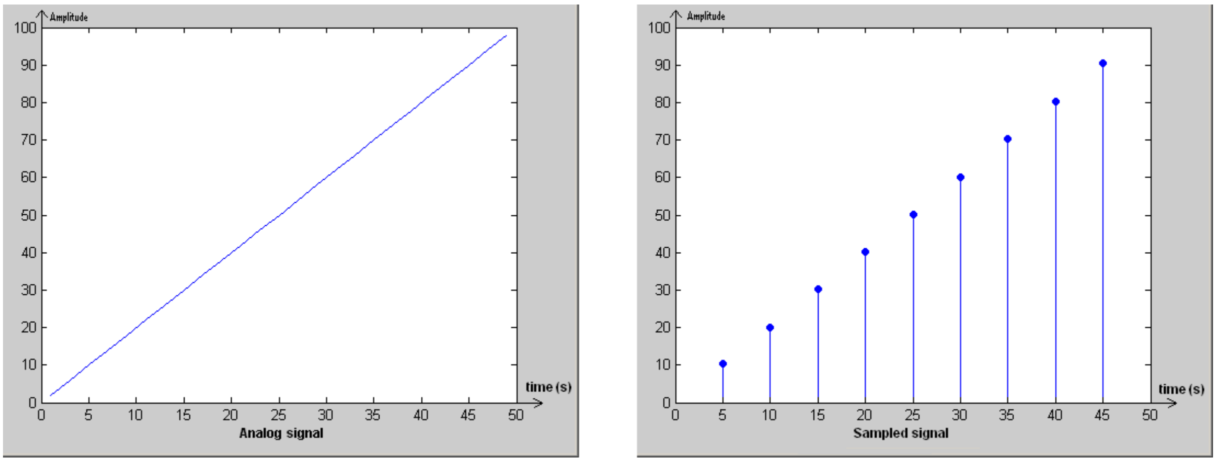
➡ only one way to store, process and transmit information units independently of their meaning is preferred

- Audio-visual signals have specific properties which require dedicated techniques for representing efficiently them:
  - data are representation of analog signals
    ➡ sampling + quantization
  - large amount of data to represent a small part of the original source of information (e.g. 1s of audio/video)

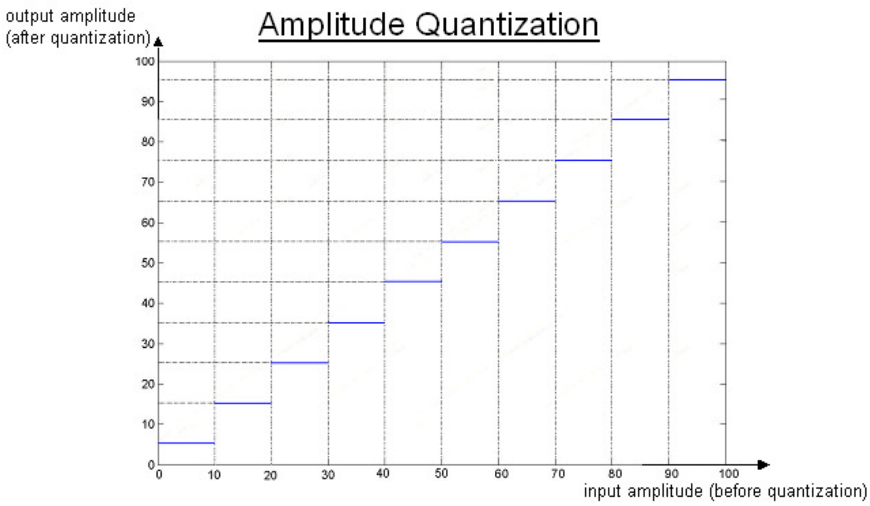
As this was mentioned in the presentation, the problem posed here is the need to greatly reduce the quantity of binary information elements in order to represent images destined for storage or transmission. Although the images are quite varied, meaning that the information they contain may all be very different (color, light, objects), it is still preferable to put in place a unique procedure for information compression encoding, which can be applied to each image, independent of its characteristics.

Furthermore, audiovisual signals are analog (continuous) signals whose representation requires a very important quantity of data (for example for a single second of video footage, you need to be able to store a sequence of 25 images). The effective representation of such signals needs adequate methods to be used.

The binary representation of an analog signal is only possible if the signal has undergone sampling (classically temporal, spatial, etc.) so that one can work on a finite number of samples from this signal. The amplitudes linked to an analog signal also stretch over a continuous interval that will need to be discretized according to different criteria: this is signal quantization. These two stages are presented here briefly for a 1D signal but they will be examined in more detail in the sequel to this module for a 2D image signal.



The figure above represents a ramp type signal before and after sampling. The sampling frequency equals 0.2Hz ($\frac{1}{5}s^{-1}$). The amplitude of this signal varies continuously over the interval [0, 100], so a quantification will be necessary.



The figure above represents an example of quantification: if a sample of the signal has an amplitude included in the interval [10k ; 10(k+1)[, « k » being a natural integer, the amplitude value would be brought by quantification to 10k+5.

After quantization, the data have a discrete representation that it will be possible to encode. Typically we would use binary encoding.

Large amount of data for representing audio-visual information

- Data size requirements
  - **One Teletext page = 25 rows × 40 char => 1 Kbyte**
  - **One A4 text page = 50 rows × 80 char => 4 Kbytes**
  - **One second of telephone (300 Hz - 3400 Hz) => 8 Kbytes**
  - **One second of stereo hi-fi audio => 176.4 Kbytes**
  - **One A4 fax => 1.1 Mbytes**
  - **One image (color-slide quality: 2000 × 3000) => 18 Mbytes**
  - **one second of TV program (video signal) => 21 Mbytes**
  - **one second of HDTV program => 230 Mbytes**

Need data compression

To illustrate the problem that a large quantity of data are required to represent a little information from an image signal, the document above gives an indication of the data sizes required to store and transmit different images, that we regularly find in our environment. These use Anglo-Saxon units, so remember that 1 byte = 1 octet = 8 bits.
For example, a single faxed sheet of A4 paper needs a capacity of 1.1 Mb to transmit the basic image. These different examples show how important it is to make data compression encoding before sending or storing an image.

Contex

- Need to mix in the same document different types of information
  - Text (common ; intrinsically structured ; semantic level)
  - Data tables (common ; intrinsically structured)
  - Graphics (vectorized data ; structured information)
  - Audio (list of many temporal samples ; low level representation (non structural & non semantic))
  - Image (list of scalar/vector samples issue from a scanning of 2D plane ; low level representation (non structural & non semantic))
  - Video (set of Image data along the time axis; same properties as Image data)

Apart from the fact that a large quantity of data are required to represent a little information from an image signal, the representation of this signal is also made difficult but the very nature of the image signal. This signal mixes many and varied types of information. Consider that an image may contain text boxes, data tables, pictures and so on.
The "Compression size/Quality" compromise also directly depends on the image content. If we consider for example a geographical map accompanied by a legend, it is not possible to simply recognize shapes and the borders of the various continents, seas and oceans, once decoding has occurred. The writing must not be too deteriorated so that legends and names on the map can still be easily read by any user.

Main aspects of image compression

- Type of images to compress ? which services ?
- Image quality:
  - lossless image coding
  - or lossy image coding

    in that case: with or without visible distortions?
- ***Compression rate / Quality***
- Codec complexity : coder end , decoder end
- Error sensitivity (error robustness):

  effects of transmission errors on the decoded image
- Fixed or variable compression rate:

  $\Rightarrow$ ***transmission requirements***

Before applying data compression encoding to an image, one must ask several, often trivial question but which have an important influence on the results obtained after compression:

- What type of image are we dealing with? What will it be used for?
- Do you need an exact encoding (reversible but highly constrained which gives a low compression rate)? Otherwise, will any deterioration be visible to the naked eye (here again, the compression rate is affected)?
- The compression rate must be as strong as possible to save sufficient storage space and transmission time as possible, but as that affects the quality, what type of balance are we striving for?
- A rise in the compression rate increases the decoder complexity and consequently the calculation cost for the system. Here again a suitable compromise must be found depending on the intended usage.
- How well does it need to stand up to errors? Transmission errors may have strong, direct repercussions on the decoded image.
- Will the compression rate be fixed or variable? What will the transmission conditions be like?

When it comes to the crunch, the required usage of the encoding is as important as the encoding itself. Before starting, you must define the compression rate and depending on your objectives, choose the compression size/quality ratio that you want to obtain.

In Summary

✠ **Very « hot » applications**

· huge volume of information

· wide spread of applications and services

digital photography (MMDB)

↓

HDTV

✠ **Multiple dependences**

· which images (≈ types & services)

· which resolution, frame rate/s, ...

· which quality required (*acceptance level*)

· which communication network

✠ **Multiple requirement scalabilities**

(multiscale, multiple qualities ...)

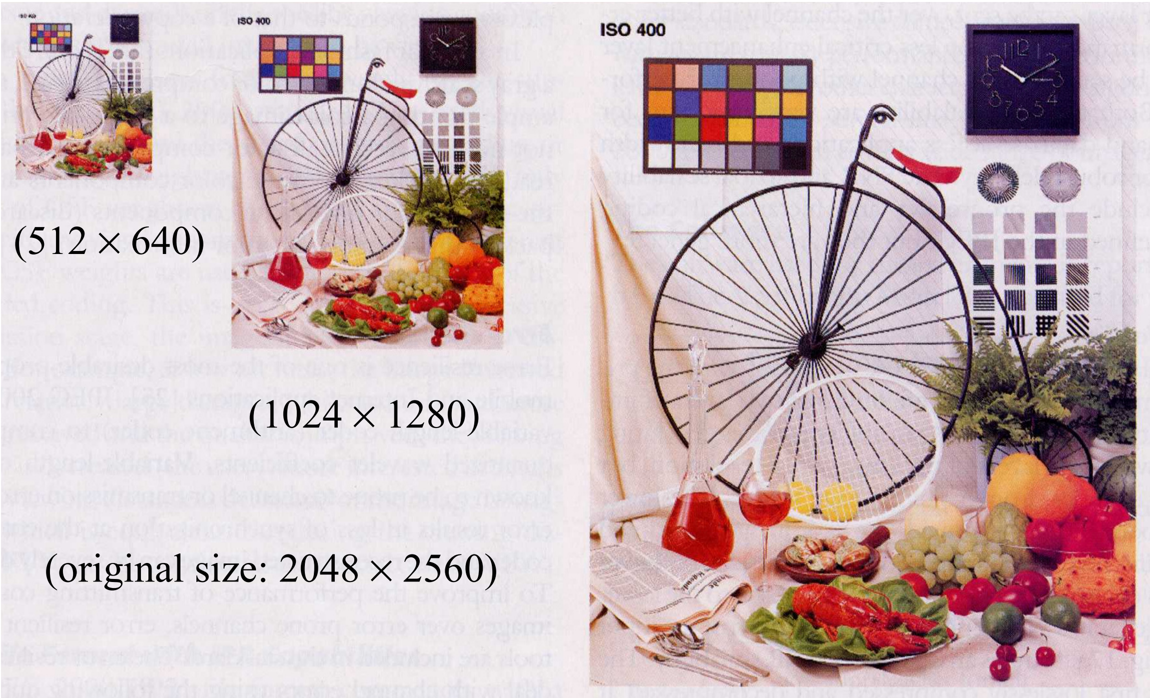
■ Other functionalities than pure coding: object manipulation,...

To sum up, what we have put forward leads to the conclusion that setting up image compression encoding is a complex procedure. On the one hand, you must consider a very large volume of information and a wide range of applications and services. On the other hand, the encoding itself will depend on a number of parameters: the image being processed (type, content, usage), its resolution, the quality required for the intended usage and the accepted amount of deterioration, the transmission network to be used, and so on. In addition to these difficulties, there may be complexity, linked to features other than the encoding itself (object manipulation, etc.).

The next section presents the case of encoding for a high definition image; by its nature this type of image creates a constraint that has a direct effect on the compression complexity.

Example of HD image: Image *Bike*

(decoded at 3 scales from the same encoding process)



The figure above presents a high-definition (HD) image, "Bike" which, after compression encoding, has been decoded to 3 resolution levels and different sizes, but using the same process. The presentation is in a pyramid form, from the highest resolution (to the left of the picture) to the finest (on the right). The image with the highest resolution appears to be sharp, and colors and details are clearly rendered. Deterioration does not seem to be perceptible by the human eye in this case. However, for an HD image such as "Bike" with a 2048×2560 resolution, new compression encoding problems appear.

Note: be careful in relation to the representation given on the illustration above, because if the display does not keep the same resolution as the original, there may be under-sampling.

Problems

- Digitalization of audiovisual signals
  - specific problems with HD images / video
    - image sensors (multi-spectral sensors, resolution, sensitivity, high speed luminance capture)
    - quantization (high precision for audio / some imagery systems, high speed conversion)
- High data compression rate but quality saving !

  for storage and for transmission (reduced bandwidth)
- Others requirements
  - multiple adaptations (transmission rate, scalability, errors robustness, packetization)
  - access to partial contents, content description, …

JPEG2000, MPEG4, MPEG7 standards

When we digitize audiovisual signals, problems specific to HD image appear:

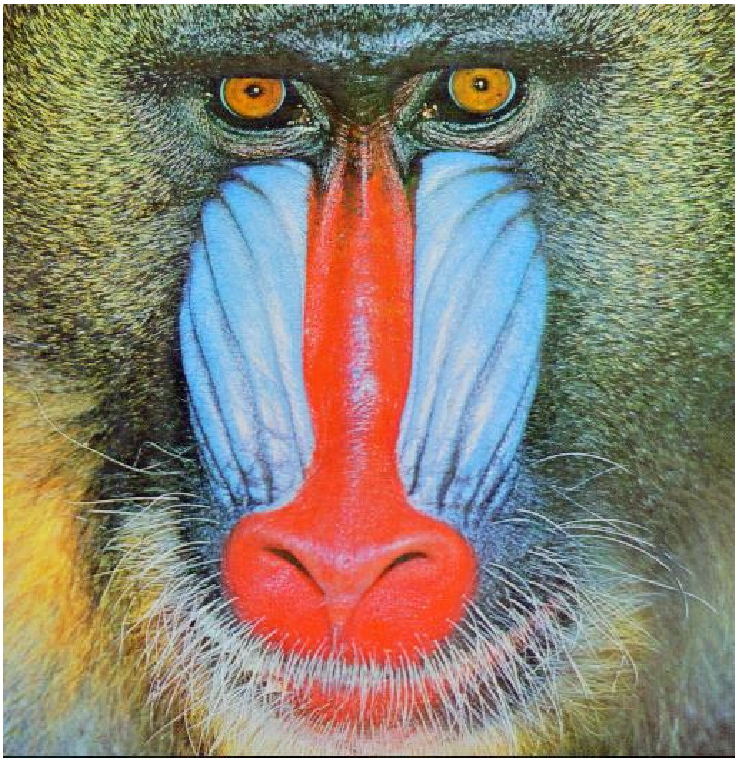
- the type of capturing device used (multi-spectral types, resolution, sensitivity, high lighting speed capture, etc.)
- The way in which quantification is chosen.

Furthermore, we are seeking to attain high compression rates, considering the intended applications. However, as this is a high definition image, we wish to keep a high degree of quality. The compromise between compression rate and quality is limited. Finally, by taking into consideration other constraints, such as the necessity to adapt the encoding (according to error resilience, transmission mode, etc.), it has become necessary to create compression standards that may meet the different constraints caused by these problems and which we can apply to images independent of their nature. The most common standards for pictures and video are JPEG, GIF, JPEG2000, and MPEG4.
The next section presents an illustration of different results obtained for a picture called "Mandrill" which was encoded in JPEG with different parameters and compression rates.

Example of JPEG coding : original of *Mandrill*

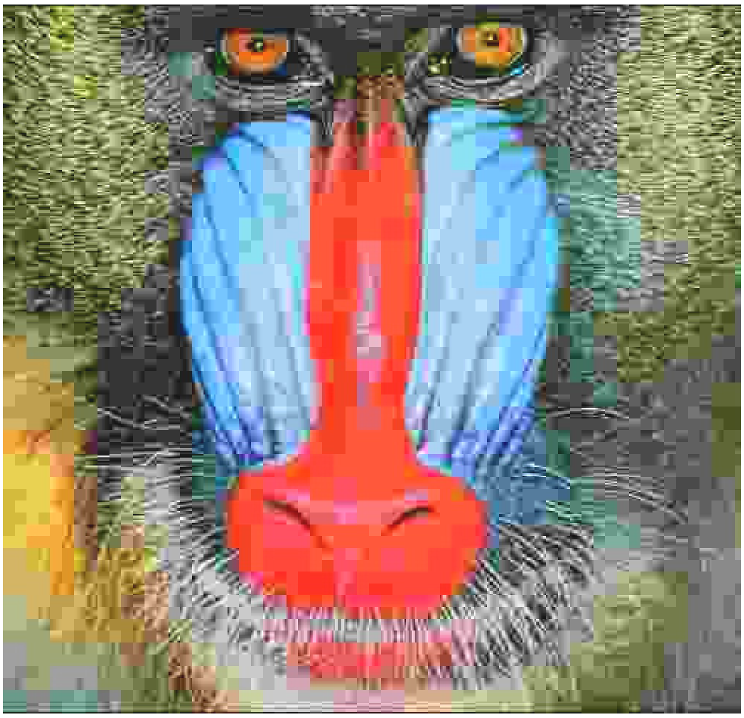
Original:
512×512
$\Rightarrow$ 24 bpp

$\Rightarrow$ **768 Ko**



Above is the original "Mandrill" picture with a size of 512×512 pixels. A pixel is coded on 24 bits (three times 8 bits because the initial representation is in full RGB colour, so there are 2^{24} possible levels for each pixel). The image is thus represented by a volume of $512 \times 512 \times 24$ = 768 Ko. We are going to put this image through a series of JPEG encodings, with different parameters, to compare the visual quality of the results and the compression rates obtained.

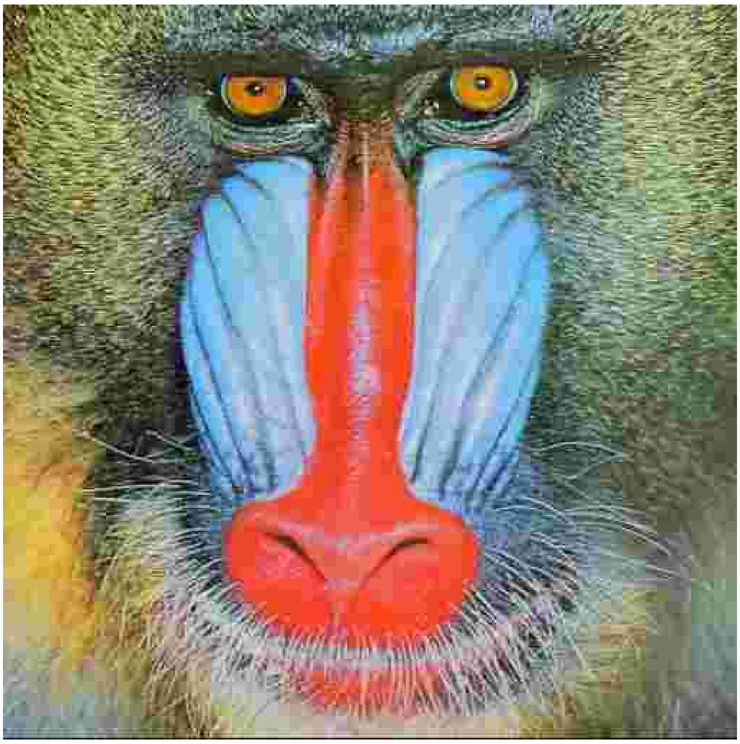
Example of JPEG coding (q = 5 ; 9 Ko)



τ_C =768/9
≈ 85

Above is the "Mandrill" picture that has undergone a JPEG encoding. Quantization was carried out with a low quality factor q (q=5). The compression gain is relatively high. The size of the data describing the image is only 9Kb. There is a high compression rate τ_C, worth 768/9=85. However, the image has lost much of its visual quality. Colors and shapes are rougher than on the original image and a blockiness effect quite clearly appears over the whole image (blocks of pixels are represented by practically the same color).

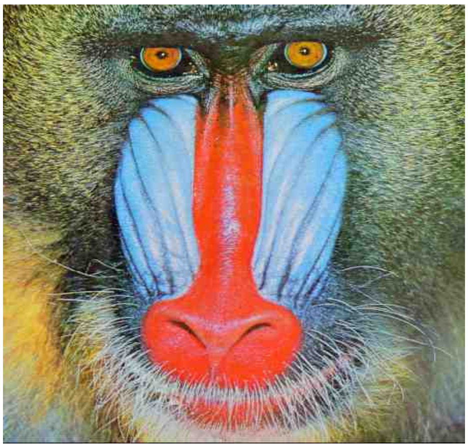
Example of JPEG coding (q = 10 ; 17 Ko)



τ_C =768/17
≈ 45

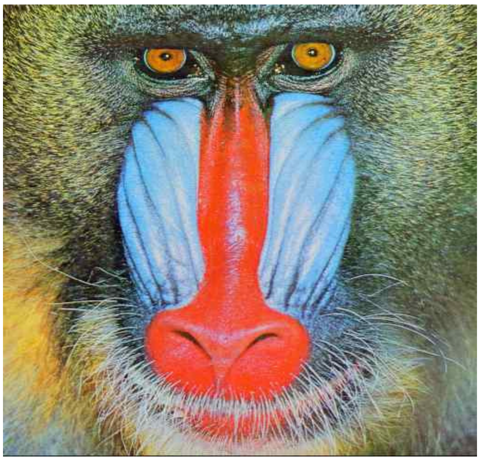
Another example of a result with a higher quality factor, although it remains quite low (q=10). The compression rate is lower than the previous example (τ_C = 45). The image quality has improved and the blockiness effect is less apparent, although it can still be seen on the vivid reds and blues of the baboon's face. The range of colors is lower than the original image, but it gives a better representation of the monkey's fur than the previous encoding. Note that by doubling the quality factor q, the compression rate is divided by 1.88, which is nearly half.

Example of JPEG coding (q = 25 ; 32 Ko)



τ_C =768/32
≈ 24

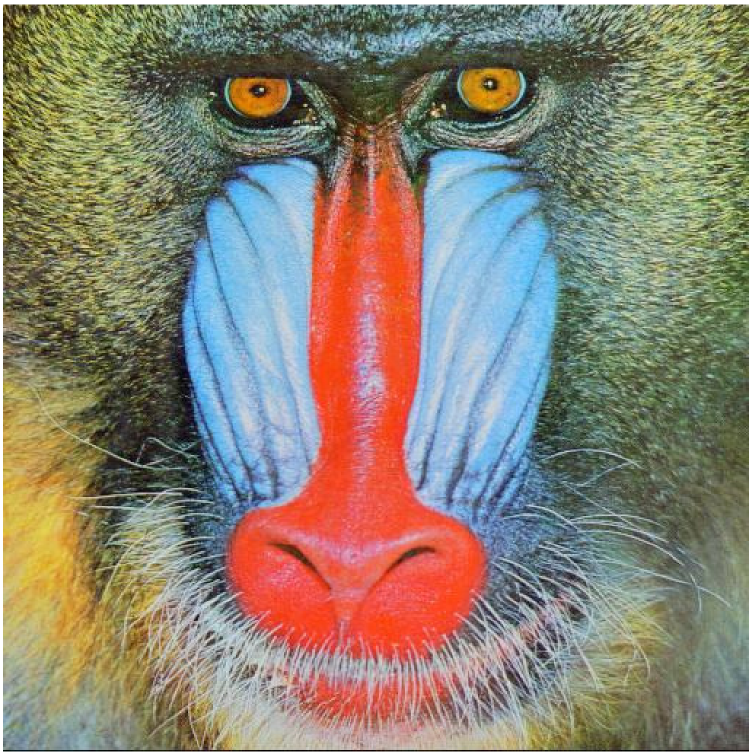
Example of JPEG coding (q = 35 ; 40 Ko)



τ_C =768/40
≈ 19.2

Two more results obtained by giving precedence to image quality rather than the compression rate. However, this time, the compression rate was divided by only 1.25. The evolution of the compression rate according to the quantization is generally not a linear function.

Example of JPEG coding (q = 75 ; 76 Ko)



τ_C =768/76
≈ 10.1

Finally, the image above presents a result where the image quality is paramount over the compression. The number of bits required to store the encoded image is 10 less than what is required to store the original image. It is however difficult for the human eye to detect the loss of picture quality: it is hard to differentiate between this and the original image in terms of the color range and the sharpness of colors and textures.

Here we have presented image processing under its different aspects. It was created from new requirements made by information technology, and in turn uses those same technologies to ensure its rapid development. With its expansion, four wide domains of application have appeared: restoration, encoding, analysis and image synthesis.

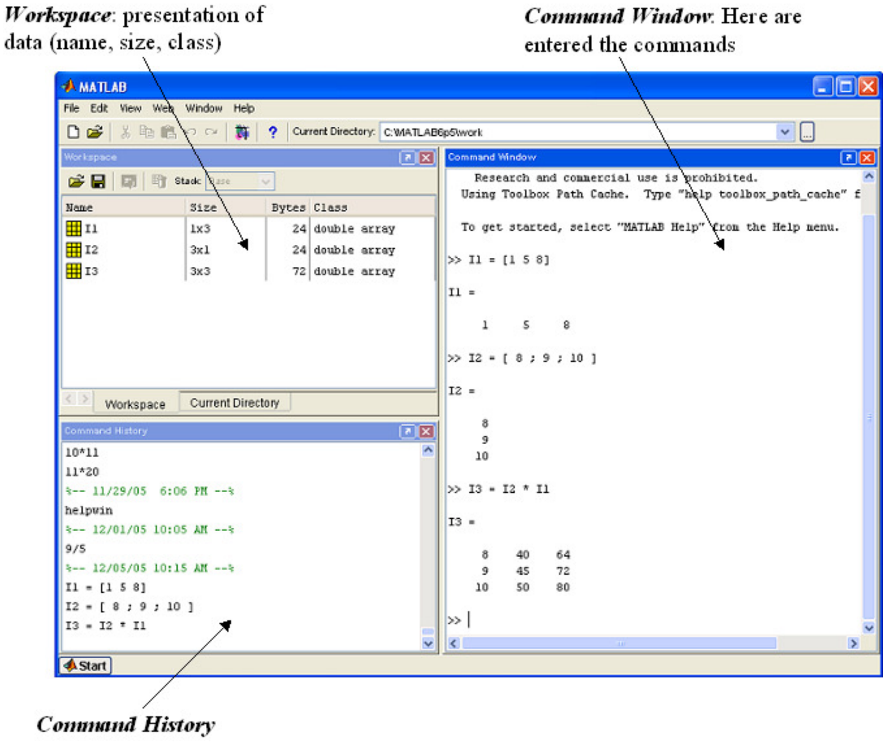
In this course, the domains of analysis and encoding have been accompanied by examples of concrete applications, in order to give an initial view of the possibilities and results offered by the various image processing.

However, in the real world, the images that surround us cannot generally be processed directly and they are not digital. They need to be digitized in order to be processed with adapted algorithms and systems. This digitizing stage will be looked at in the next resource (Image digitization).

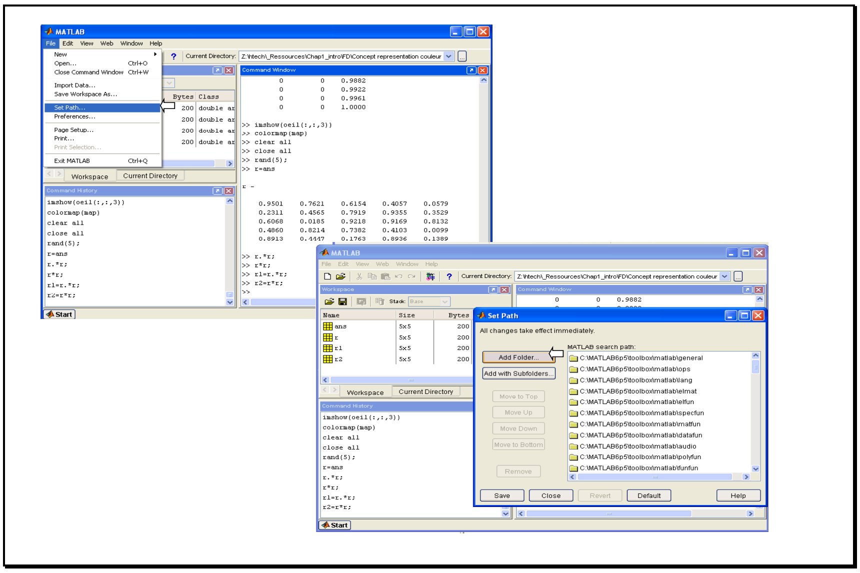
Exercise Chapter 1– Introduction to Matlab

MATLAB is a high-level scientific calculation language and an interactive environment for developing algorithms, visualizing and analyzing date or carrying out numerical calculations.

When you start Matlab, by default you find yourself with a console, a workspace and a command history. An editor is also available by using a simple ***edit*** command.



ACTION: Start Matlab and add your own working folder paths to the path list in the path browser (don't forget the subfolders). The ***pwd, cd, ls*** Unix commands are available.



*<u>Note</u>: when a user runs a command in the console, Matlab will go looking in the folder that you indicated in the path browser; if a **function** or a **script** matches that command, the first one found will be the one used (so take care with the folder order and the names of scripts and functions).*

Exercise 1 – Operators

Between each exercise, we recommend that you use the ***clear*** and ***close all*** commands so that you can empty the workspace and close all the figures.

1.1 – Enter the command *a=[1 4 5 ;4 8 9]*, what does this command correspond to?
In the console, enter the *a=rand(5)* command. Then enter the ***help rand*** command to get a description of the operation carried out by the ***rand*** function. Finally, enter the command: *a=rand(5)* followed by a semi-colon « *;* »
What difference can we observe in the console with the *a=rand(5)* command? Deduce from t his the role of the « ; »in Matlab.

1.2 – In Matlab, the « : » operator is very useful. It can be used, among other things, to swap elements from a row or a column of a matrix.
A word of caution: the index 0 does not exist in Matlab. the first element of a matrix is accessed by an index of 1. For example *array(1,1)* for images accesses the value of the pixel (1st row, 1st column). The first index is for the rows, the second index for the columns.
To fully understand these concepts, try the commands:

- *a(2,1)*
- *a(1:3,1)*
- *a(:,2)*
- *a(1, :)*
- *a([1 4], :)*
- *a(1 :2 :5, :)* (the *1:2:5* command sweeps the interval [1,5] by steps of 2)

Be careful however not to put a « ; » at the end of a row to visualize the results obtained in the console.

1.3 – Matlab is an interesting tool for matrix calculations. The various classical matrix operations (addition, multiplication, eigenvalues etc.) are there of course but there are also element by element operations that are very useful in image processing; these are available through a specific operator (for example: « **. *** », « **. /** »).
Enter the following commands (without « ; » at the end of the line to see the results):

- *a= [0 0 0; 1 1 1; 2 2 2]*
- *b=a+4*
- *c=a*b*
- *e=a.*b*

Explain the difference between « ***c*** » and « ***e*** ».

ACTION: Create a matrix *A* sized 4×4 using the method of your choice (use ***rand*** or enter the items one by one). How can you access:

- the first row of *A*?
- the fourth column of *A*?
- the first three items of the fourth row in *A*?

Exercise 2 – Display

Matlab also offers a wide range of diverse and practical display possibilities. For example, you can plot function on a logarithmic scale or view the values of a matrix as an image.

2.1 – A vector can be displayed using the ***plot*** command. Enter the ***help plot*** command to obtain more information about this function and on functions with similar properties (***subplot***, ***semilogx***, etc.).

Try out the following commands:

- *x=1:3:10*
- *plot(x)* then *plot(x,'r')*
- *y=rand(4,1)*, then *plot(y)*, then *plot(x,y,'g')*. Interpret the difference between these two commands.

The ***plot*** function is very useful for obtaining for example the curves of different plane functions.

2.2 – Displaying a matrix on the other hand corresponds to displaying an image. Each item (n, m) in the matrix is considered as having the value of the pixel (n, m) with which it is associated. Check this by entering the following commands:

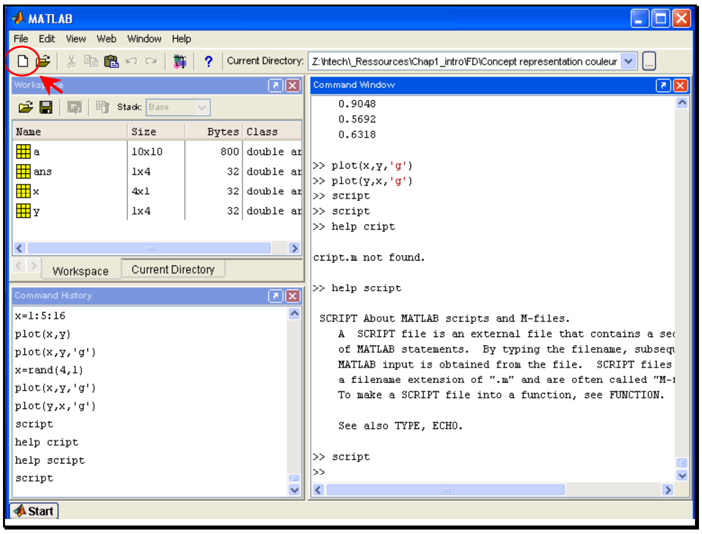
- *a=rand(10)*10 ;* (so that the elements are not limited to [0,1])
- *a=exp(a) ;* (to obtain larger spans between the items of vector a)
- *image(a)*

Images can be displayed with the ***image***, ***imagesc***, and ***imshow*** functions. Try zooming with the mouse; what kind of interpolation is used?

Exercise 3 – Writing scripts

The classic extension of a MATLAB file is .m. We can find two types of .m files: function files (which we will look at later in the chapter) and script files, which are a set of commands that can follow on from each other. Using script files is a way of saving your commands from one Matlab session to the next. To open a script file:

- either type the ***edit*** command;
- or click: file ⇒ new ⇒ M-file;
- or click the "blank page" icon.



To run a script:

- either run a ***file_name*** command in the command window (without the .m extension), making sure that the path list is consistent.
- or select lines from the .m file in the edit window and press the F9 key (a practical way of running only part of a script).

ACTION: Enter all the commands from the section 1.2 into a script and save it (for example: ***test.m***). Run the script in the console and using the F9 key.

Exercise 4 – Data types

During this series of exercises, we are going to be using the ***Image Processing*** **toolbox**. This contains a large number of existing functions (do not hesitate to look in the help using the ***help*** or ***helpwin*** commands), as well as demonstrations. Sometimes we will use our own functions and sometimes we will be using those from the toolbox. However, you need to be careful with the data types. Classically and by default in Matlab, everything is a matrix of ***double***, but most of the functions in the *Image Processing* toolbox work with the ***uint8*** type to represent pixel value. You will need to convert the type each time this proves necessary (you can easily see the type of your data in the workspace).

ACTION: From the image base, download the '*FRUIT_LUMI*' image into your working folder. Read and display the image file respectively using the commands ***imread*** and ***imshow***:

fruit = imread('FRUIT_LUMI.bmp');
imshow(fruit);

By consulting the workspace, look at the size and data type. So as to display a sub-image that corresponds to the top left corner of 64×64 pixels, test this command:

imshow(fruit(1:64,1:64));

The matrix *fruit* now represents the '*FRUIT_LUMI*' image. Try adding this matrix directly to any number (e.g.: *fruit+8*). The console returns an error message:

Function '+' is not defined for values of class 'uint8'

This is because the '+' operator is defined only for "double" type elements. To carry out this operation, you will need to force the "double" type for the elements in the matrix by typing: *double(fruit)*. Try again after this conversion.

Exercise 5 – Writing your functions

Use the function files as you would in any classical imperative programming situation. These are also .m files. To use a function, you must call it with call parameters either in a script or in another function.

ACTION: Use the ***template.m*** file to write your own max_min function that will find the difference between the largest and smallest element in a matrix. You can use the Matlab *max* and *min* functions. Be careful to save the name of your function (e.g. ***max_min)*** in a file of the same name (example ***max_min.m***).

Contents of the ***template.m*** file:

```
function[out1, out2] = template(arg1, arg2)

%--------------------------------------------
%
%    Description:
%
%    Input:
%           arg1:
%           arg2:
%
%    Output:
%           out1:
%           out2:
%
%    Date:
%    Modif:
%---------------------------------------------
```

Note: You can use a function directly in the command window (providing that you use the correct paths) or within a script or another function.

Solution to the exercise for the introduction to Matlab

The first objective of this exercise is to make you familiar with Matlab if you have never used it before and to remind current users of its basic functions.

1 – Operators

1.2 –

The command *a=[1 4 5 ;4 8 9]* returns the matrix 2×3: $\begin{pmatrix} 1 & 4 & 5 \\ 1 & 8 & 9 \end{pmatrix}$.

The command *a=rand(5)* returns a matrix 5×5 made up of random values between 0 and 1.
Finally, in Matlab, the operator « ; » is used when you do not want to display a command’s result in the console. This operator is useful, for example when you work with large matrices (such as images), which are sometimes long to display and often not very representative of the data.

1.3 –

Working with the operator « : ».

1.4 –

In Matlab There are two types of matrix operations that use the « * » and « / » operators:

- matrix multiplication and division,
- element by element multiplication and division (joint use with the operator « . ».

The command *c=a*b* will perform a matrix multiplication of matrix « a » by matrix « b » : $c_{ij} = \sum_k a_{ik}.b_{kj}$.

The command *e=a.*b* will perform an element by element multiplication of matrix « a » by matrix « b » : $e_{ij} = a_{ij}.b_{ij}$. The matrices have to be the same size.

ACTION:

Create a matrix *A* with a size of 4×4 by directly entering these coefficients one by one: A=[1 2 3 4;5 6 7 8;9 10 11 12;13 14 15 16]

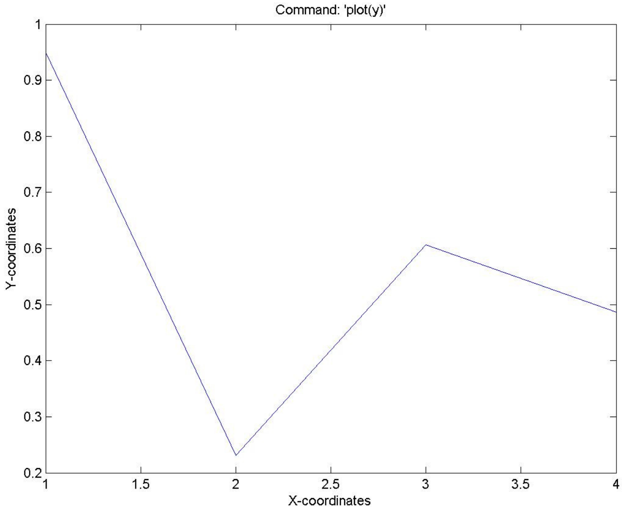
$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}$$

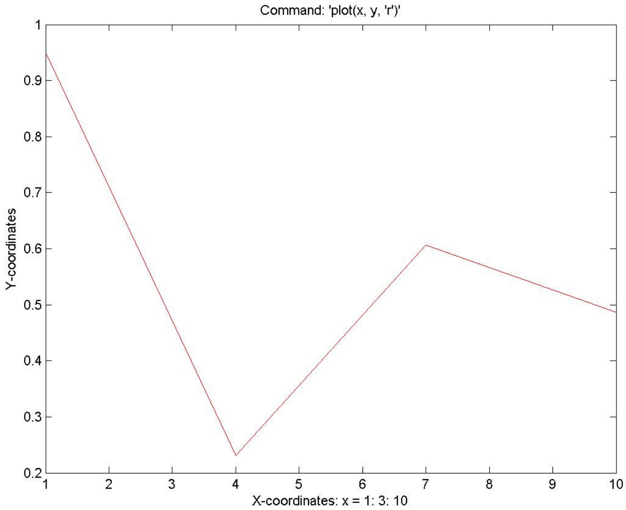
The 1st row of A is given by the command: *A(1,:)*.
The 4th column of A is given by the command: *A(:,4)*.
The first 3 elements of the 4th row of A are given by the command: *A(4,1:3)*.

2 – Display

2.1 –

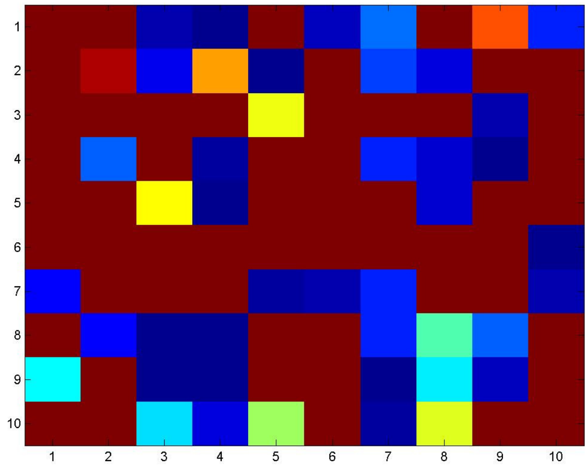
The command *y=rand(4,1)* returns a vector of 4 elements. By typing *plot(y)*, a curve appears. This curve represents the evolution of the vector « y » depending on the indices of the vector elements. However it is possible to modify the abscissas by creating an abscissa vector « x » in a given unit of the same size as the vector « y »: you plot y according to x by the command *plot(x,y)*.





2.2 –

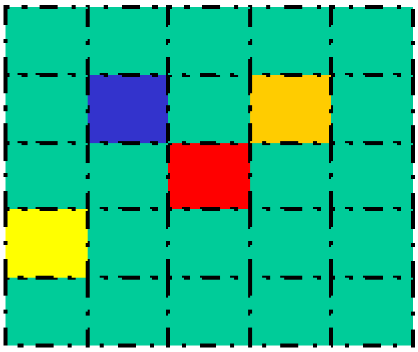
Here is an example of the result obtained by typing the commands indicated with a display by *image(a)*.



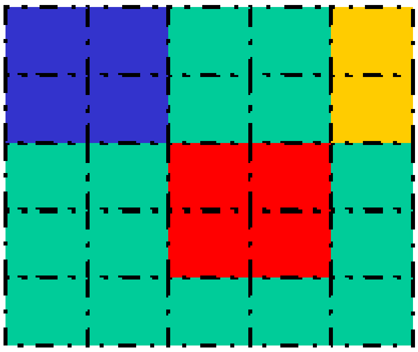
The 10×10 sized matrix is represented here by 10×10 square pixels of different colors. The colors correspond to the values of the various elements in the matrix.

The zoom uses a zero-order interpolation of a nearest neighbor type. This means that it is a simple imitation of an image pixel by several pixels on the display screen.

Let's consider for example a screen of 5×5 pixels, on which a 5×5 image is represented. The screen pixels correspond to the represented grid.



The figure below presents a ×2 zoom around the red pixel located in the centre in the case of a nearest neighbor interpolation.



The red pixel is simply « duplicated » to twice its height and width on the screen pixels.

3 – Writing scripts

Working on a script.

4 – Data types

Working with uint8 type data.

5 – Writing your functions

The Matlab « ***max*** » function (and respectively the « ***min*** » function) given an M×N matrix as input, returns a vector of size N of which each element e_k is the maximum element (and for ***min*** the minimum element) of the matrix column k.

Here is the solution function:

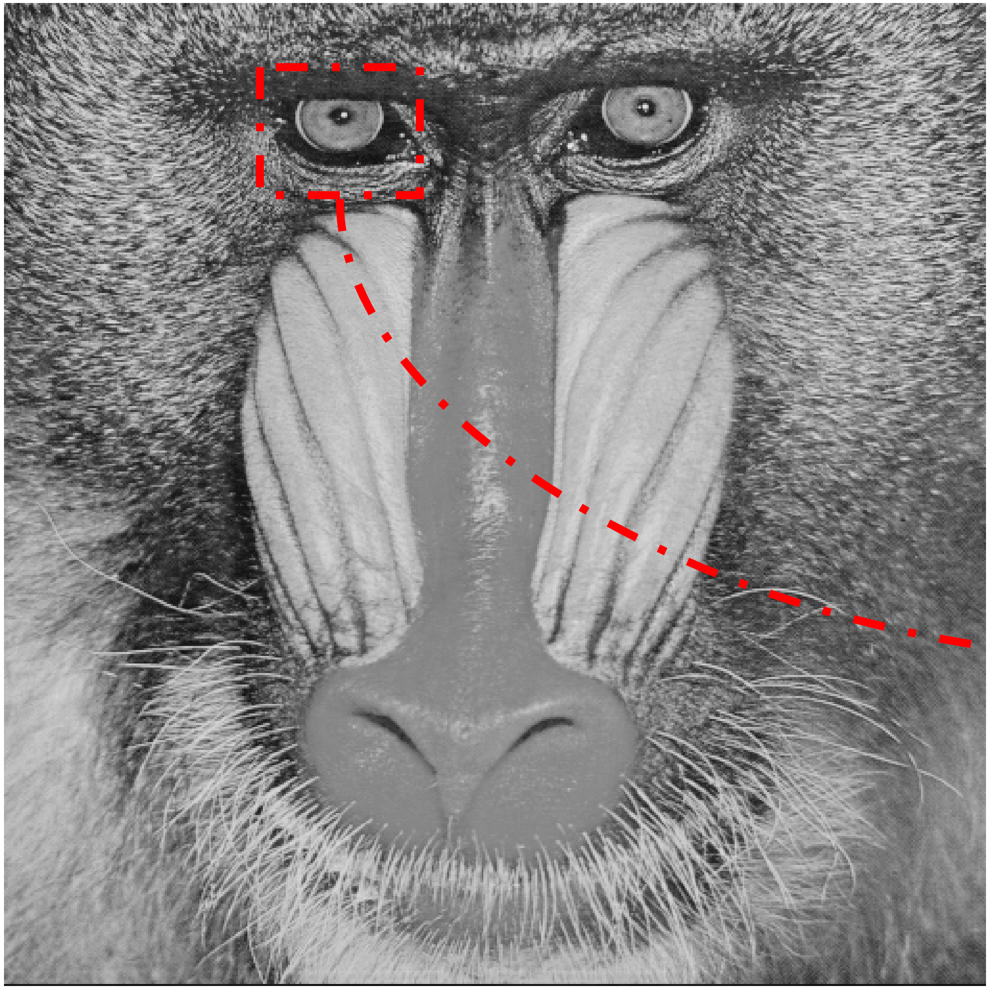
```
function[out] = max_min(A)

%-------------------------------------------
%     Description: difference between the max and min
%     elements of a matrix A corresponding to a monochrome
%     image
%
%     Input:
%           A: the matrix on which the search occurs
%
%
%     Output:
%           out: the value of the difference
%--------------------------------------------

out = max(max(A))-min(min(A));
```

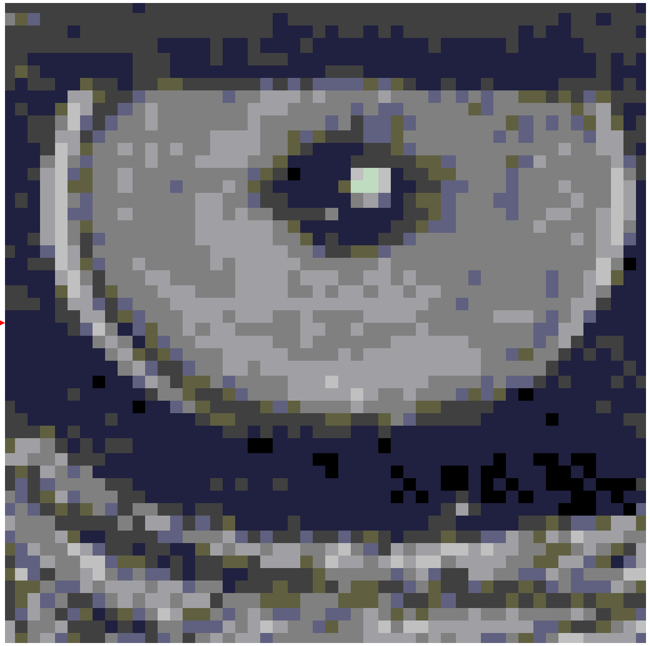
How is composed a grayscale image?

Grayscale image Mandrill (512 × 512 pixels)



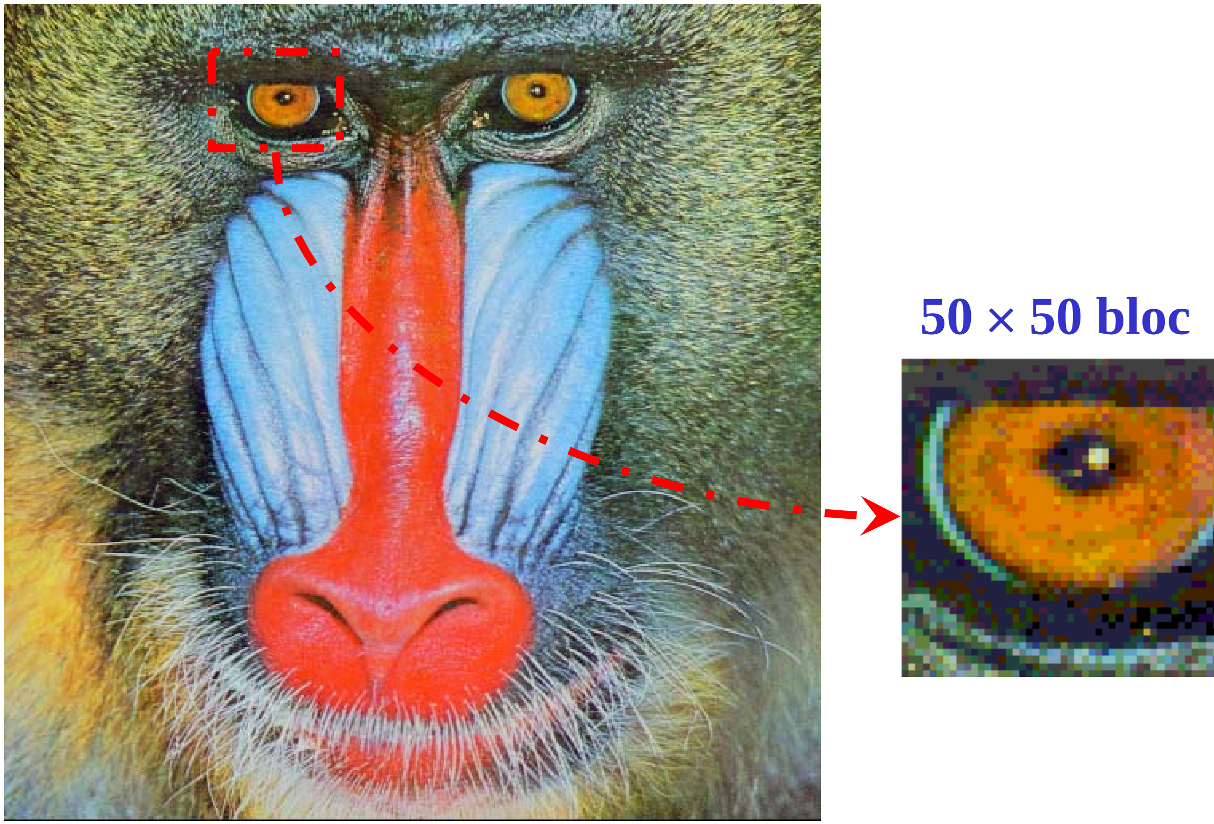
Zoom of a 50 × 50 pixel bloc

⇒ Zoom for visualizing the luminance levels (256 possible levels)

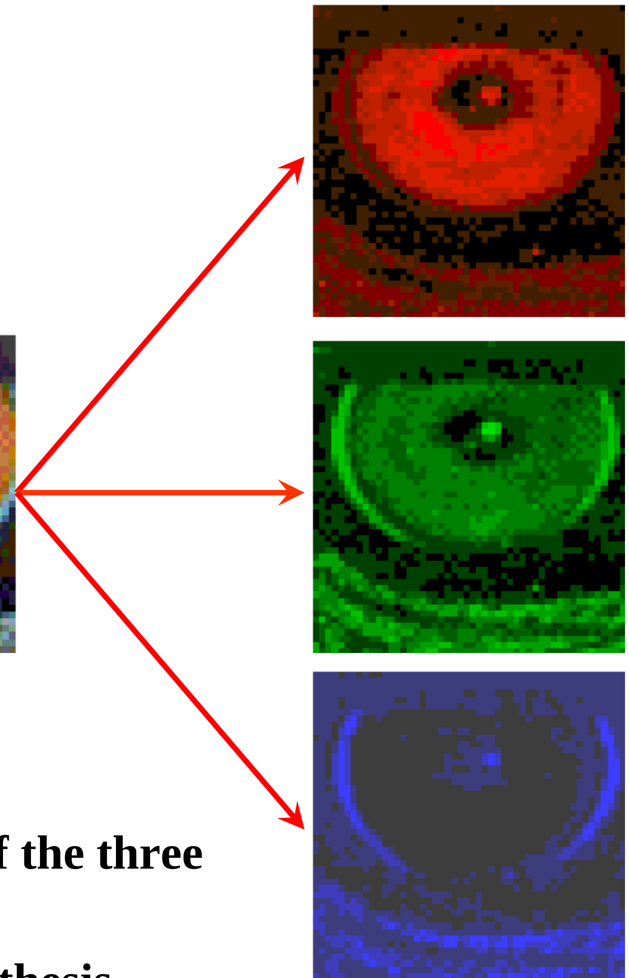


How is composed a color image?

Color image Mandrill (512 × 512)



The RGB Planes



- **On a screen, the color is based on additive synthesis of the three components: RED - GREEN - BLUE**

Note : printing for full color documents by subtractive synthesis, (components : Yellow, Cyan, Magenta + Black).

Exercise Chapter 1 – Breaking up a color image

A color image is made up of 3 components. In chapter 1 we have seen 2 color spaces to represent this: a decomposition using Red, Green and Blue and a decomposition using Luminance, Chrominance1 and Chrominance2 (there are also others). We are going to study at the Red Green Blue color space representation in Matlab.

Retrieve the *CLOWN* and *CLOWN_LUMI* images. Create a folder and put the images in it. Update the path lists in the path browser.

1 – Difference between a grayscale image and a color image

Create two variables « *im1* » and « *im2* » in which you are going to load respectively the *CLOWN* and *CLOWN_LUMI* images (using the ***imread*** function). Visualize these two images using the ***image*** function. Check their type and the size of the associated data; what differences do you notice?

2 – Viewing the Red Green Blue planes of a color image

The variable « *im1* » is a three-dimensional table of a N×M×P type. In other words, for a color image, N×M represent the number of pixels in the image (N points per row, M rows) and P represents the Red Green Blue planes. For a color image, we will have P=3 and for a grayscale image P=1.
Create 3 new variables *imr*, *img*, and *imb* in which you should have respectively the red component (P = 1), the green component (P = 2) and the blue component (P = 3) of the *CLOWN* image. Visualize these three planes.

Important note: To view the 3 RGB planes, which are the contents of the variables *imr*, *img*, and *imb*, you will need an appropriate LUT (LUTs, or Look-up Tables are presented in chapter 2).
Three LUT have been created and placed for this purpose in the files called *lutr.m*, *lutg.m*, and *lutb.m* for viewing, respectively, the red plane, the blue plane and the green plane. Put the *lutr.m*, *lutg.m*, and *lutb.m* files into your working folder. Once you have displayed one of the three planes of the *CLOWN* image, taking care to select the window in which this plane appears (the active window for Matlab), run the *lutr* command in the console for the red plane, *lutg* for the green plane and *lutb* for the blue plane.

Solution to the exercise on breaking up a color image

The aim of this exercise is to show you on a concrete example of how to carry out Red Green Blue (RGB) additive synthesis on a color image. Matlab uses this RGB decomposition for displaying colors so it is an ideal tool for viewing the different planes.

1 – Difference between a color image and a grayscale image

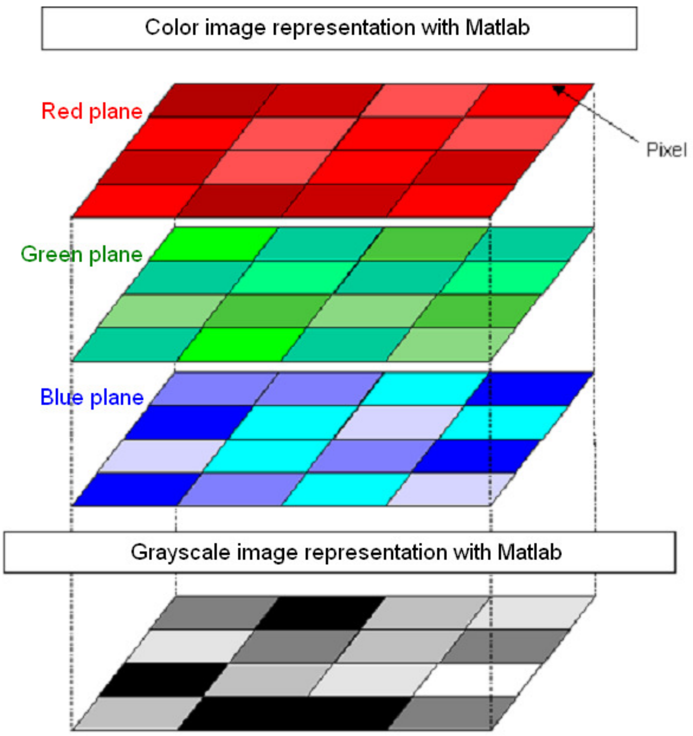
To acquire the *CLOWN* and *CLOWN_LUMI* images in the variables *im1* and *im2* you must type these commands:

```
im1=imread('CLOWN.BMP') ;
im2=imread('CLOWN_LUMI.BMP') ;
```

To display them, type:

```
image(im1) ;
image(im2) ;
```

Looking at the workspace, you can see that *im1* (color image) is a three-dimensional table (512×512×3) while *im2* (monochrome image) has only two dimensions (512×512). The size in bytes of *im1* (786432 bytes) is also three times larger than that of *im2* (262144 bytes).
The first two dimensions of tables *im1* and *im2* correspond to the number of points per row and the number of rows for the image. The image *CLOWN,* whether it is displayed in scales of gray or in color, is an 512×512 image, so it is normal that the first two dimensions are identical for the variables *im1* et *im2*. The third dimension of *im1* indicates that we are on the Red, Green or Blue plane. This dimension is thus equal to 3. For a grayscale image, represented only by scales of gray varying over [0,255], this component is worth 1 and so is not indicated in the workspace. We conclude that a color image corresponds to the superposition of three single color RGB images.



2 – Visualizing the Red Green Blue planes of a color image

Here is the script for acquiring the three RGB planes in the variables *imr*, *img*, and *imb*:

```
imr=im1(:,:,1);     % Red plane
img=im1(:,:,2);     % Green plane
imb=im1(:,:,3);     % Blue plane
```

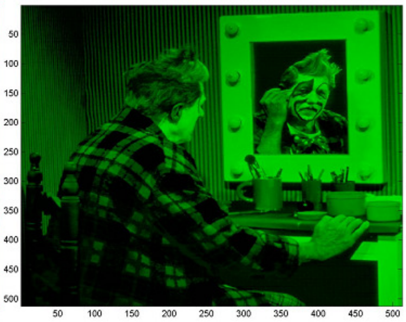
Once you have imported the *lutr*, *lutb*, and *lutg* files into your working folder, here are the scripts for displaying the three planes of the image:

```
% Displaying the Red plane
figure      % Create a new window for displaying the image
image(imr);
lutr;

% Displaying the Green plane
figure
image(img);
lutg;

% Displaying the Blue plane
figure
image(imb);
lutb;
```

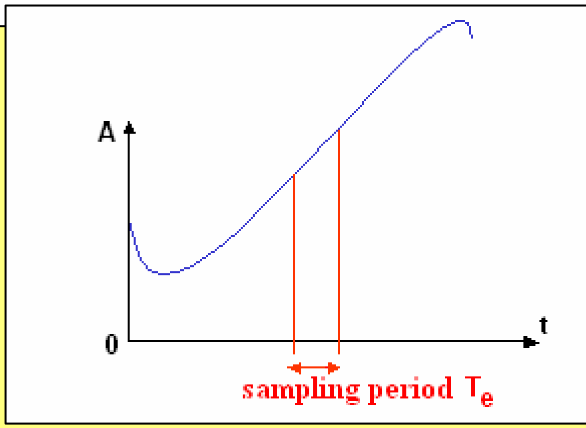
Here are the obtained results:


From an analog signal to a digital signal:
3 steps

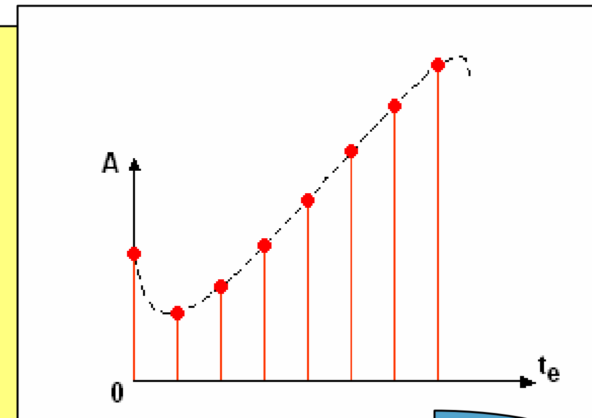
Digitalization of a temporal signal (1-D)

Continuous signal (reference)



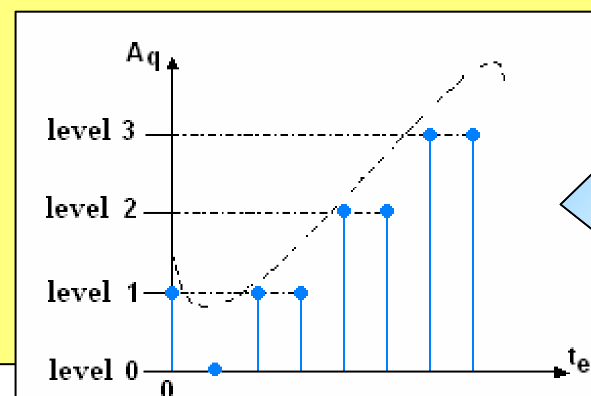
Step 1

Sampled signal



Step 2

Quantized signal (4 levels)



Step 3

Coding (4 levels ⇒ 2bits)

	CODING	
	Bit 2	Bit 1
level 3	1	1
level 2	1	0
level 1	0	1
level 0	0	0

From an analog signal to a digital signal: 3 steps

- **Sampling :** the original temporal signal is represented as a set of finite values. The values are sampled with the sampling period T_e.

- **Quantization :** the continuous signal amplitude is represented by a set of discrete values (reconstruction levels).

- **Coding :** the binary coding of the reconstruction levels uses k bits ($\Rightarrow 2^k$ possible levels).

IN CASE OF IMAGES

- The images are 2-D signals. The sampling is performed along the spatial dimensions « x » and « y » (not along the time axis as previously).

- Typically the number of luminance reconstruction levels is 256, each level is thus coded by 8 bits (natural binary coding).

Chapter 1

MULTIMEDIA SIGNAL PROCESSING

Image digitalization

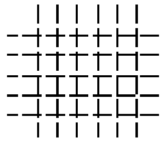
Sampling and Quantization

Sampling and Quantization

- Computers handle discrete data.

- Sampling

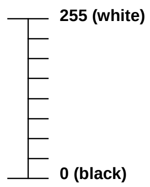
  - Sample the value of the image at the nodes of a regular grid on the image plane.

  - A pixel (*picture element*) at (m, n) is the image intensity value at grid point indexed by the integer coordinate (m, n)

- Quantization

  - Is a process of transforming a real valued sampled image to one taking only a finite number of distinct values.

  - Each sampled value in a 256-level grayscale image is represented by 8 bits.

In the previous resource (Image processing – Examples of applications) we briefly mentioned on the fact that digitalization implies a double discretization:

- Firstly from the domain of the image definition: the 2D spatial domain for still images (respectively the 2D spatial domain and time for moving images): this is ***sampling***.
- Secondly from the image signal amplitude: the luminance signal in the case of monochrome images, the three color signals in the case of color images (representing “Red Green Blue” (RGB) or "Luminance, Chromatic1, Chromatic2": Y, Cr, Cb): this is ***quantization***.

We can only then speak of digital (or digitalized) images once this double discretization has been carried out, in space and in amplitude. The basic (or canonical) representation of an image corresponds to a 2D table, in which each element corresponds to a pixel. For a monochrome image, each pixel is encoded on 8 bits, for example, and in this way could take 256 different values (effect of the quantization). In the case of a color image, the pixel will have three components that will represent the “Red Green Blue” (RGB) components or "Luminance, Chromatic1, Chromatic2" (YCrCb)) according to the chosen model.

<u>Note</u>: if R, G, B each take their value from [0, 255], and Y too, the two chromatic components Cr and Cb take their initial value from [-128, +127]. In practice, these are encoded by adding an offset of 128 so they have both a range dynamic of [0, 255].

A Typical Image Processing Scenario

Recall: What is an Image?

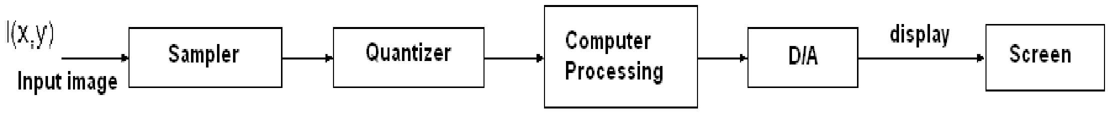
– A function I(x,y) over two spatial coordinates of a plane

– To obtain digital data for digital processing:

• *Sampling (spatially) and Quantizing the luminance values*

• *Can be done by single chip i.e. Charge-coupled Device (capteur CCD)*

Typical image processing scenario



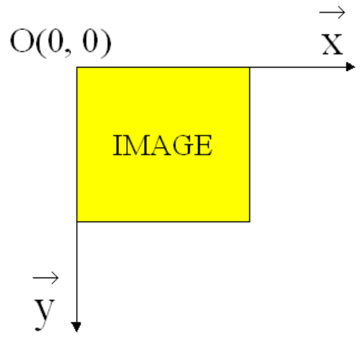
The figure above shows the complete chain used to carry out any type of image processing:

- The image is represented by a function of two continuous variables, these variables corresponding to spatial coordinates (x, y).
- The first function is *Sampling*: we sample the continuous image signal (discretization of the spatial coordinates).
- The result is then injected into the *Quantization* function: for a monochrome image, we often choose 256 luminance levels.
- The image is now digitalized, so we can apply the required image processing (*cf. Processing*), which could be binarization, contrast enhancement, etc.
- The processed digital image is finally sent to a digital-to-analog converter (DAC) to obtain a signal that can be displayed on a monitor.

The typical functional chain for image processing involves an operation for digitalizing the images that we will then process. This operation is sampling followed by quantization. These two notions were reviewed in connection with signal processing and electronics. However the design and achievement of these two stages in imaging are quite different because images are 2D spatial signals and their representation for color images requires us to consider to digitalize the three components.

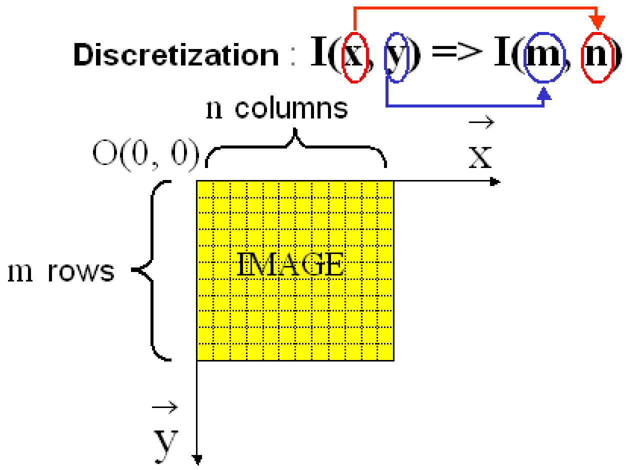
Two important remarks:

1 – The input signal I(x, y) is a 2D image. In images, the axis (Ox) and (Oy) are represented and oriented as shown below:



The (Oy) axis is directed from the top of the image towards the bottom.

2 – Once these two axis are discrete, we obtain « m » rows and « n » columns:

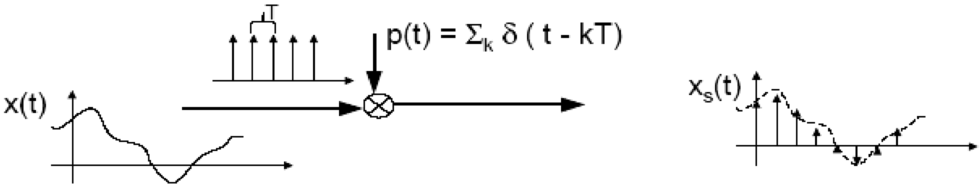


In image processing, **we need to speak of an image I(m, n)** and not I(n, m). **The coordinate order is modified** because « m » corresponds to the discretization according to (Oy) and « n » according to (Ox).

Recall: 1-D Sampling

- **Time domain**

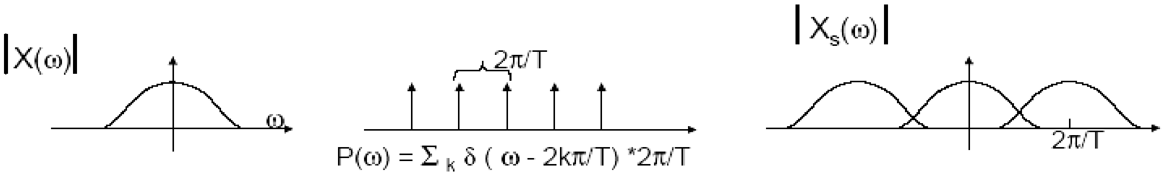
  - Multiply continuous-time signal with periodic impulse train



- **Frequency domain**

  - Duality: sampling in one domain ⇔ tiling in another domain

    - the Fourier Transform of an impulse train is an impulse train (proper scaling & stretching)



Before moving on to sampling a 2D signal, we should recall some results about 1D signal sampling. The most widespread type of sampling in electronics and signal processing is time-based sampling. The figure above presents the various results issued by such processing in the time and frequency domains.

Note: $\omega = 2\pi f$, with ω: pulsation in rad.s^{-1} et f: temporal frequency in Hz

- In the time domain: starting from a signal « x » depending on time, you must choose a sampling period « T ». The sampled signal $x_S(t)$ will thus be represented by its values at instants t = T, t = 2T, t = 3T, … Practically, to create such a sampling, you must carry out the product of convolution of the analog signal x(t) with a series of impulses p(t). These impulses correspond to the sum of the Dirac pulse (Dirac delta function) at time kT, k being an integer.

- In the frequency domain: there is a duality between the time domain and the frequency domain. The fact of sampling a signal in one of these domains causes the periodization of the same signal in the other domain. Here X(ω) (resp. P(ω)) represents the frequency spectrum of x(t) (resp. p(t)). The sampling period T in the time domain has thus created the periodization (period 1/T) of the spectrum in the frequency domain (see the representation of $X_S(\omega)$).

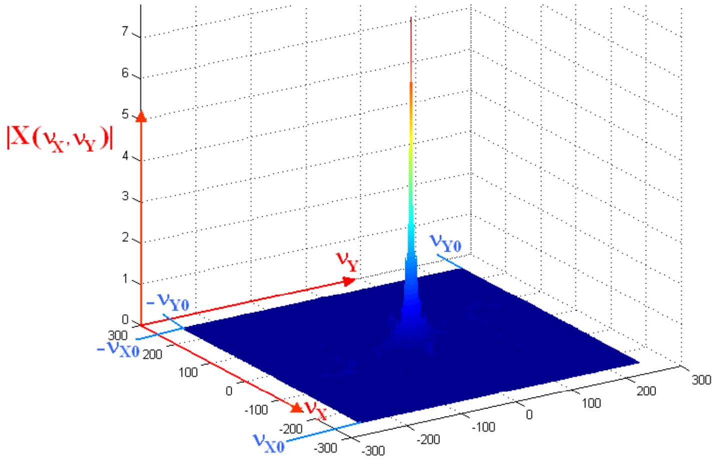
This duality between the two domains can create aliasing effects due to spectrum overlapping, so certain constraints must be met.

Extension to 2-D sampling (1)

- Bandlimited 2-D signal

  - its Fourier transform is zero outside a bounded region is spatial frequency domain: $|\nu_X| < \nu_{X0}$ et $|\nu_Y| < \nu_{Y0}$

Example : the image *Barbara* and its spectrum

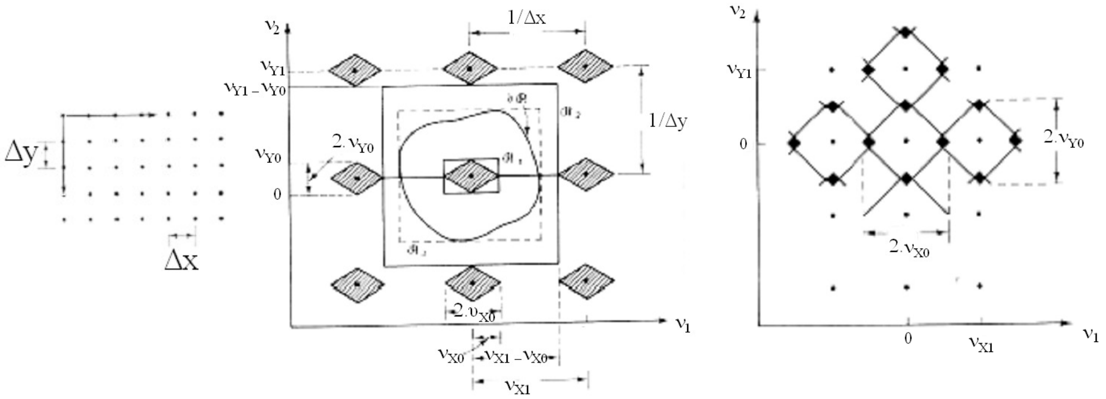


The problems caused by the sampling of two-dimensional signals are most likely similar to those of one-dimensional signals. The Nyquist criterion is applied in the same way. However, an interpretation in a two-dimensional space is necessary to better understand the phenomena linked to sampling, such as the effects on the sampling pattern geometry.

The figure above represents an example using a Fourier transform from a 2D limited bandwidth function (image *Barbara*). In this case we can work with a bounded spectrum. The spectrum spreads over a spatial frequency domain defined by two basis vectors.

Extension to 2-D sampling (2)

- 2-D Comb function
  - $\mathrm{comb}(x, y; \Delta x, \Delta y) = \sum_{m,n} \delta(x - n.\Delta x, y - m.\Delta y)$
  - $FT \Rightarrow COMB(\nu_X, \nu_Y) = \mathrm{comb}(\nu_X, \nu_Y; 1/\Delta x, 1/\Delta y) / \Delta x.\Delta y$
- Sampling vs. Replication (tiling)
  - Nyquist rates ($2.\nu_{X0}$ et $2.\ \nu_{Y0}$) and Aliasing



The variations according to $\vec{x}$ (Δx) and the variations according to $\vec{y}$ (Δy) are going to define the plane on which the 2D image is located. The sampling structure is regular (figure on the left) so the arrangement of sampled points (pixels) is regular in the plane (x, y). The position of the point at coordinates (m, n) is given by: nΔx + mΔy.

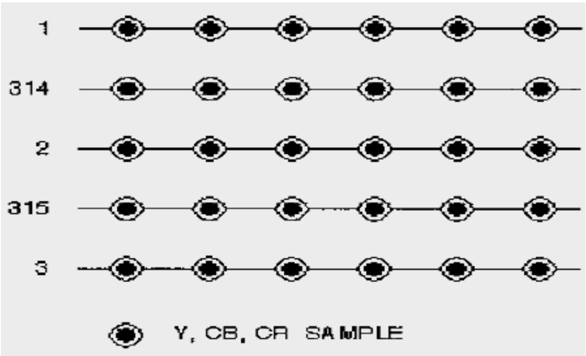
In the frequency domain, points are defined by their "frequential" coordinates ν_X et ν_Y. From a function *f*(x, y), to which we associate the amplitude spectrum F(ν_X, ν_Y,), we will obtain after sampling:

- $f_e(x, y) = f(x,y).\sum_m \sum_n \delta(x - n.\Delta x,\ y - m.\Delta y)$
- $F_e(\nu_x, \nu_y) = \left(\dfrac{1}{\Delta x.\Delta y}\right) \sum_m \sum_n F(\nu_x - n.\Delta\nu_x,\ \nu_y - m.\Delta\nu_y)$

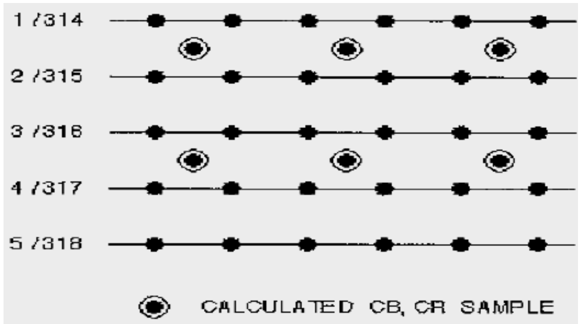
For a bounded spectrum image signal, we can reconstruct the original (non-sampled) image, from the ideally sampled image if the original signal spectra do not overlap (central figure in opposition with the image on the right). To do this, we use an interpolator R of constant spatial frequential gain (spatial frequencies), equal to ΔxΔy in the F domain and zero outside of F, so that this has no overlapping with any of the translates of F. We then have the ideal two-dimensional linear interpolator: the Nyquist spatial interpolation filter.

Sampling Lattice in Color Images and Videos

4 : 4 : 4 Sampling Structure



4 : 2 : 0 Sampling Structure



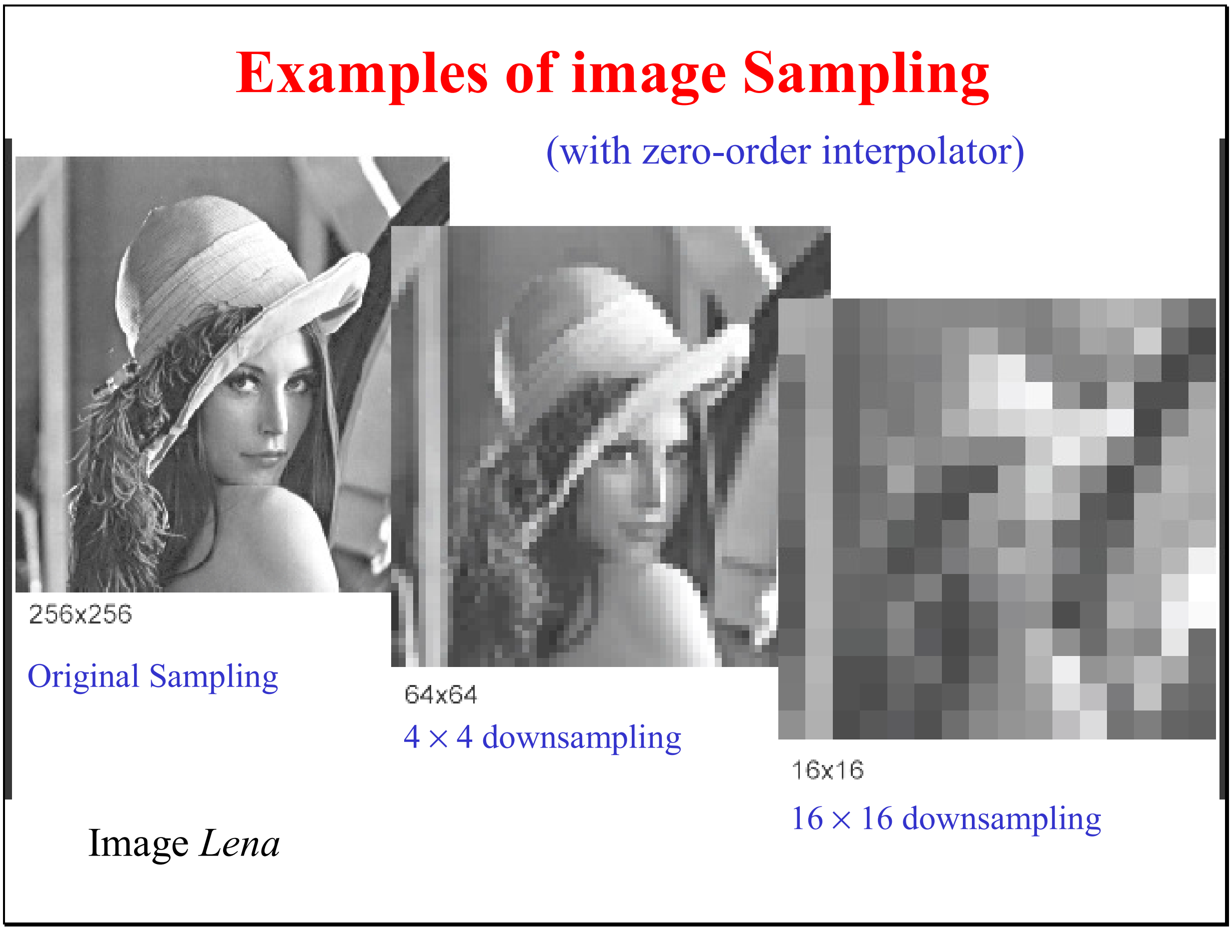
Orthogonal sampling structures

The figure above presents two common structures for orthogonal sampling of color images:

- A 4 : 4 : 4 structure: For each pixel, we have a luminance component and two chromatic components, Cr and Cb. In this case there is no sub-sampling of the chromatic components.

- A 4 : 2 : 0 structure: For each pixel, we have a luminance component (Y) but the two chromatic components, Cr and Cb are only for a group of 2×2 luminance pixels. The chromatic resolution is divided by a 2 factor along the two spatial dimensions. This format (used in DVDs) reduces the vertical and horizontal resolutions.

For these sampling structures, we transformed the RGB representation of the image into the YCrCb representation. The reduction in the spatial sampling frequency of the two chromatic components is made possible by the fact that, in natural images, the two chromatic components have a spatial frequency bandwidth much narrower than that of the luminance component.



In this figure, we have represented the image "*Lena*" sampled with two different sampling structures. The image on the left is the reference image (spatial dimensions: 256×256 pixels). The second image is sample with a sampling frequency four times lower for each of the two spatial dimensions. This means it is 64×64 pixels. For display purposes, it has been brought to the same size as the original using a zoom. This is in fact an interpolation of zero-order (each pixel is duplicated 4×4 times, so that on the screen it displays a square of 4×4 identical pixels). A pixelization (staircase) effect clearly appears, but it is still possible to distinguish the different elements in the image (the face, the hat etc.). The third image (on the right) shows the result after a downsampling of 16 in relation to the original, following the two spatial directions. Again in order to see it at the same size, we have used zoom factor 16 (interpolation of zero-order creating for each sample a reconstruction by 16×16 pixels of the same amplitude). The image is strongly pixelized and we can hardly distinguish the objects in the scene.

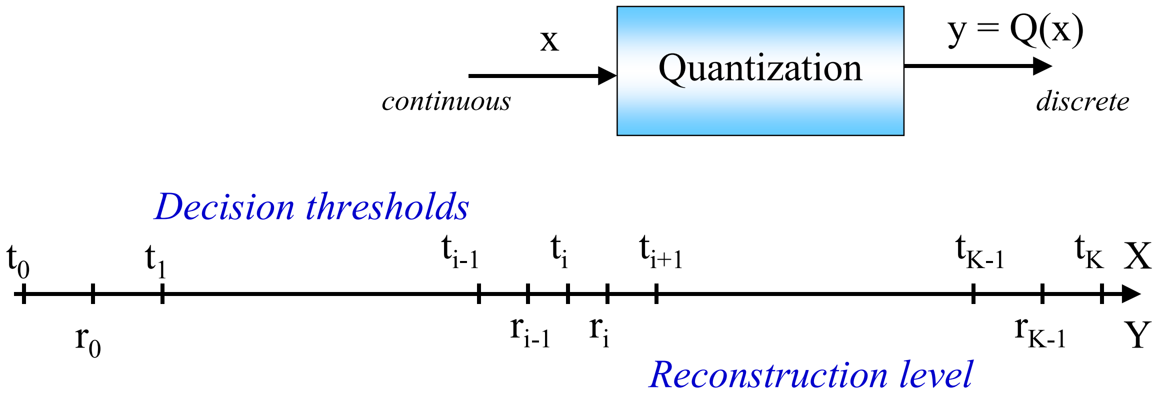
Now that we have looked at the two-dimensional sampling of a 2D signal (in this case an image), we now have to determine how to proceed with the quantization of the sampled image in order to get a digital image.

Video and Image digitalization

Quantization

Quantization

- **Scalar quantization**



- **Quantization function**

  for $t_j \leq x < t_{j+1}$ then $\quad y = r_j \quad \forall j = 0, \ldots, K-1$

  and $\quad t_0 = x_{min}\ ; t_K = x_{max}$

The image is now sampled in the spatial domain. It is made up of a set of pixels. We now have to carry out a quantization so that each analog amplitude of the image signal is represented by a discrete value. We choose a set of predefined values called reconstruction levels, spread over the dynamic range $[x_{min}, x_{max}]$ of the signal x to quantize.

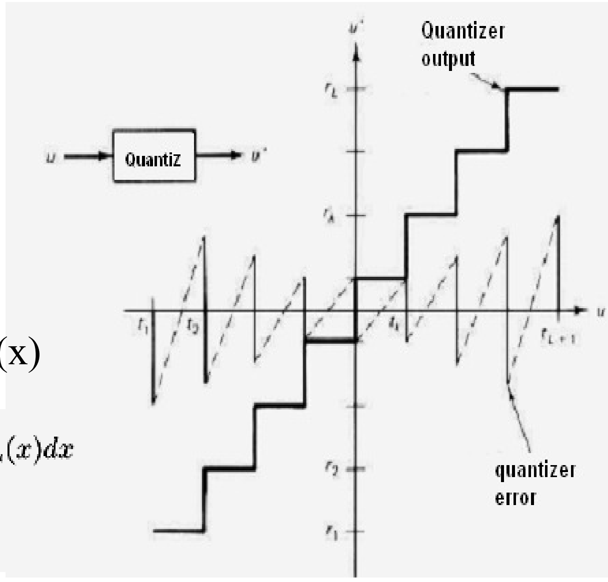
On the figure above, these reconstruction levels are defined by the sequence $(r_i)_{i \in [0,K-1]}$ made up of K values. These levels are associated with decision thresholds defined by the sequence $(t_i)_{i \in [0,K]}$ with $t_0 = x_{min}$ et $t_K = x_{max}$. Let's take for example the case of a continuous magnitude signal « x » as input into the quantizer; the quantized signal « y » output will be defined by the condition:

$$\forall i \in [0..K-1],\ \text{for } t_i \leq x < t_{i+1}\text{, then } y = r_i \text{ and } t_0 = x_{min}\ ;\ t_K = x_{max}$$

There are several ways of quantizing. The way of choosing the reconstruction levels and the decision thresholds is not necessarily a uniformly distributed type on $[x_{min}, x_{max}]$, and strongly depends on the characteristics of the image signal being quantized.

Example of a quantization law

- Multiple-to-one mapping
- Want to minimize error



- **Mean-square-error (ε)**
  - Let an input « u » with the probability density function $p_u(x)$

$$\varepsilon = E[(u-u')^2] = \int_{t_1}^{t_{L+1}} (x-u'(x))^2 p_u(x)dx$$
$$= \sum_{i=1}^{L} \int_{t_i}^{t_{i+1}} (x-r_i)^2 p_u(x)dx$$

The chart above gives an example of a quantization law. The quantizer input corresponds to « u », and the output to the signal « u' ». The decision thresholds are given by the sequence $(t_i)_i$ and the reconstruction levels are given by the sequence $(r_i)_i$. Quantization is a process that causes irreversible loss unlike certain forms of sampling that we have looked at previously. However, depending on how you configure the quantization, the human eye may not necessarily notice these errors. We must then try to minimize them. Typically, we seek to minimize the mean square error. By definition, this square error ε^2 is given by the relation:

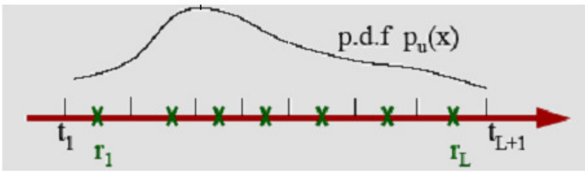
$$\varepsilon^2 = E\left[(u-u')^2\right] = \int_{t_1}^{t_{L+1}} \left(x-u'(x)\right)^2 .p_u(x).dx$$

Where: E stands for the statistical mean value and $p_u(x)$ stands for the probability density of u. By decomposing the integral, we have finally:

$$\varepsilon^2 = \sum_{i=1}^{L} \int_{t_i}^{t_{i+1}} \left(x-r_i\right)^2 .p_u(x).dx$$

Quantization must be chosen so as to minimize the mean square error but we will see that other criteria linked to the image characteristics also are interesting.

Which kind of quantization law to use? 0



- Allocate more values in more probable ranges
- Minimize error in a probability sense
  - **MMSE (minimum mean square error):**

$$\begin{aligned}\mathcal{E} &= E[(u-u')^2] = \int_{t_1}^{t_{L+1}} (x-u'(x))^2 p_u(x)dx \\ &= \sum_{i=1}^{L}\int_{t_i}^{t_{i+1}} (x-r_i)^2 p_u(x)dx\end{aligned}$$

  - assign high penalty to large error
  - square error gives convenience in math.:differential, etc.

- An optimization problem
  - **What $\{t_k\}$ and $\{r_k\}$ to use?**
  - **Last time: necessary conditions by setting partial differential to zero**

The question asked here is to know what criteria must we take into account in order to correctly choose a quantization law. The figure above represents a quantizer Q with an input signal « u », and an output signal « u' ». The decision thresholds are given by the sequence $(t_i)_i$ and the reconstruction levels are given by the sequence $(r_i)_i$. the probability density function $p_u(x)$ is also represented. In this example, the quantizer is no longer linear: the thresholds and reconstruction levels are not regularly placed. Actually, by looking carefully at the probability density of « u », we can see that the function slope is gentler (or steeper) for certain values of t_i than for others. When the slope is gentle, we can reasonably think that you need few reconstruction levels to represent the range of values described. Inversely, when the curve's slope becomes steeper, numerous values are affected in a short time and so more levels of reconstruction are needed. We have also seen that to choose a quantization law, we also need to minimize the mean square error whose expression directly depends on the parameters (t_i) and (r_i). Optimizing quantization is achieved through the choice of decision thresholds and reconstruction levels. These parameters can be directly determined by minimizing the mean square error (calculation of partial derivatives etc.).

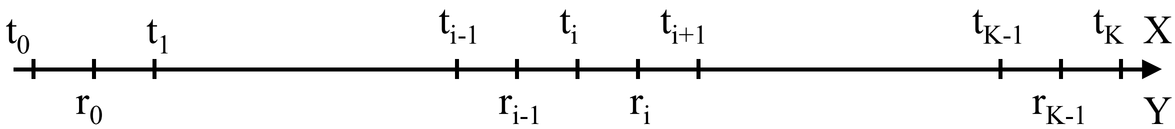
Optimal Quantization (1)

- Criteria of optimal quantization
  - Mean Square Error (y = Q(x))

$$\varepsilon^2 = E\{ (X-Y)^2 \} = \int_{-\infty}^{+\infty} (x-y)^2\, p_x(x)\, dx$$

$$\varepsilon^2 = \sum_i \int_{t_i}^{t_{i+1}} (x-y_i)^2\, p_x(x)\, dx$$

minimization of ε^2: $\begin{cases} t_{i+1} = (r_i + r_{i+1}) / 2 \\ r_i = E\{X / x \in [t_i , t_{i+1}]\} \end{cases}$



- **Special case:** uniform probability density function: $p_x(x)$ is constant then $t_{i+1} = (r_i + r_{i+1}) / 2$ and $r_i = (t_i + t_{i+1}) / 2$

The optimal quantization criteria is linked to the minimization of the mean square error. The quantizer input corresponds to the signal X, and the output to signal Y. The reconstruction levels are given here by (r_i), and the decision thresholds by (t_i). We are going to try to determine the relations ensuing from this optimization and minimalization criterion: To minimize ε^2, we calculate its partial derivatives:

- In relation to t_i : $\frac{\partial \varepsilon^2}{\partial t_i} = p_X(t_i).(t_i - r_{i-1})^2 - p_X(t_i).(t_i - r_i)^2$
- In relation to r_i : $\frac{\partial \varepsilon^2}{\partial r_i} = -2.\int_{t_i}^{t_{i+1}} p_X(x).(x - r_i).dx$

By cancelling the first partial derivative, we obtain the relation: $t_{i+1} = \frac{r_i + r_{i+1}}{2}$

By cancelling the second partial derivative, we obtain:

$$\int_{t_i}^{t_{i+1}} p_X(x).(x - r_i).dx = 0 \iff \int_{t_i}^{t_{i+1}} p_X(x).x.dx = \int_{t_i}^{t_{i+1}} p_X(x).r_i.dx$$

$$\iff E\{X; x \in [t_i, t_{i+1}[\} = r_i.\int_{t_i}^{t_{i+1}} p_X(x).dx = r_i$$

This means that minimizing the mean square error (optimal quantization) implies that the decision thresholds d_i and the reconstruction levels q_i be given by the relations: $\begin{cases} d_{i+1} = \frac{q_i + q_{i+1}}{2} \\ q_i = E\{X; x \in [d_i, d_{i+1}[\} \end{cases}$.

These are the two relations obtained by Max. These show that the optimal quantizer for a signal using a non-uniform probability density function is not a linear quantizer.
In the particular case of an input signal whose probability density is constant over [x_{min}, x_{max}], we have : $p_X(x) = \dfrac{1}{(x_{max} - x_{min})}$, and we obtain the relations:

$$\begin{cases} r_i = \dfrac{t_i + t_{i+1}}{2} \\ t_{i+1} = \dfrac{r_i + r_{i+1}}{2} \end{cases}$$

The optimal quantizer is linear in this case: $\Delta t_i = \dfrac{x_{max} - x_{min}}{K}$.

For a signal normalized over [0, 1], the minimized square error is worth: $\varepsilon^2 = \dfrac{1}{12.K^2}$.

We have seen that, in order to optimize the quantizer, we must minimize the mean square error of quantization with this criterion. Naturally, other criteria may be used, in particular those concerning the visual appearance.

Optimal Quantization (2)

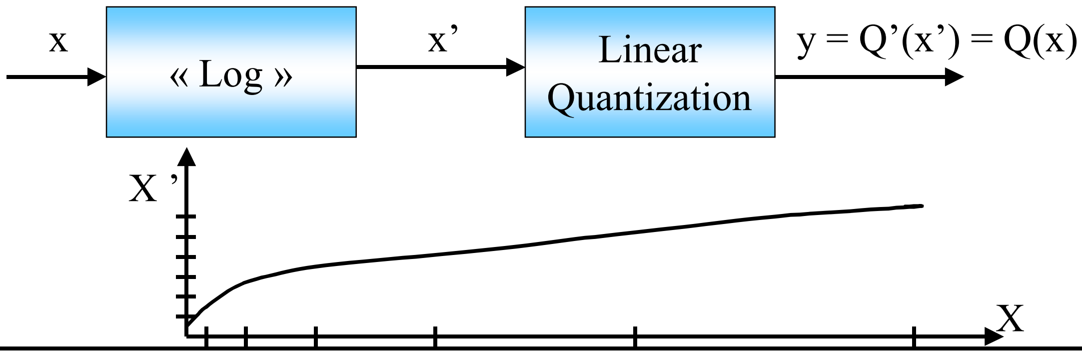
- **Maximum entropy quantization** : H(Y) maximum

$$\Rightarrow \quad p_i = \int_{t_i}^{t_{i+1}} p_x(x)\, dx = \text{constant} \quad \forall i = 1, \ldots, K$$

- **Psycho-visual quantization of images**

  - Perception of small stimuli on a background: $\Delta L / L = \text{cst}$

  - Cst $\cong$ 0.01 in luminance TV range (given by Weber law)

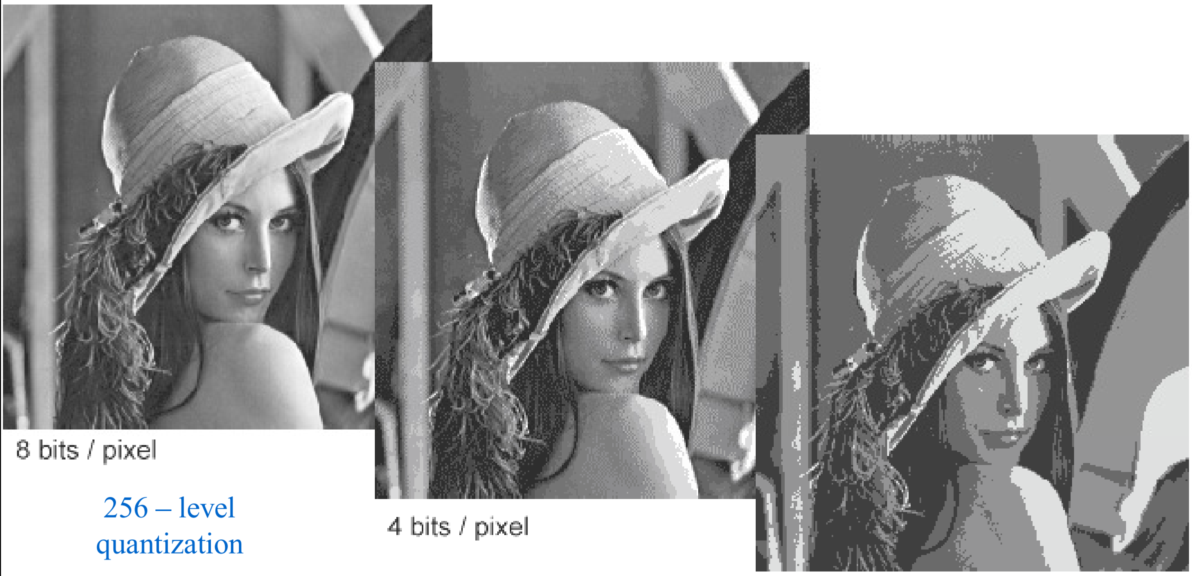


The characteristics of the human system of vision can be used to design the quantization law. For a usual stimulus of a given form and fixed ΔL amplitude, the visibility threshold $\Delta L / L$ is more or less constant (L being the luminance). In the TV luminance range, it varies slightly when L goes from black to white. For a quantization at the visibility threshold, we would have $r_{i+1}-r_i=\Delta L$. It becomes possible to use a uniform quantization on a compressed signal by a non-linearity « f » (see diagram above). You would of course need to carry out the reverse operation coming out of the quantizer with an expansion function.

This non-linear compression function is defined by $f(x) = \int_{L_0}^{x} \Delta L(x).dx \approx \lambda.Log(x / L_0)$. For television, compression « f » is already used: it is the gamma correction given by $f(x) = x^{\frac{1}{\gamma}}$ that compensates for the non-linear nature of CRT-type TV screens. We then apply a linear quantization over 256 leveks. In color television, we usually quantize the luminance and chromatic signals separately. From a perceptual point of view, the YCrCb space is more uniform than the RGB space.

We can see by looking at television-type applications that despite the interesting properties of the optimal quantizer, it can sometimes be a better idea to use a non-linear compression function and then to use a linear quantization (even if the signal's probability density function is not uniform). The "compression function – quantization – expansion function" series would be chosen in order to approximate the optimal quantization law.

Example: quantization of an image



8 bits / pixel

256 – level quantization

4 bits / pixel

16 - level quantization

2 bits / pixel

4 - level quantization

Image *Lena*

This illustration shows examples of a quantization carried out on the image Lena:

- For the image on the left: quantization is followed by a natural binary coding with 8 bits per pixel. There are $2^8 = 256$ reconstruction levels to represent the magnitude of each pixel. It is the typical case of a monochrome image (only in gray scales).
- For the middle image: quantization is carried out with a 4 bits per pixel coding, giving $2^4 = 16$ reconstruction levels. Contours are well rendered but textures are imprecise in some cases. These are areas in the signal with a weak spatial variation, which suffer more visually due to the appearance of false contours (loss on the face and the shoulder).
- For the image on the right: quantization is carried out with a 2 bits per pixel coding, so we have $2^2 = 4$ reconstruction levels. The deterioration seen on the previous image is even more flagrant here.

We have now seen the various steps of image digitalization: double discretization by spatial sampling and then by quantization. The images are now digital and are ready to be processed with appropriate techniques, according to the required application.f

Chapter 1 – Introduction to digital image processing

TEST

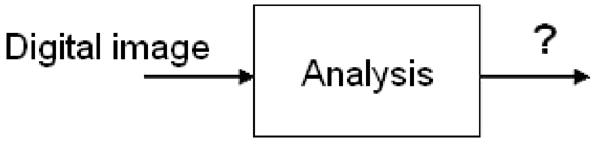
1 – In the table below, use YES and NO to indicate whether the ***Objective*** given is in relation with the image processing ***Domain*** indicated.

Domain / ***Objective***	Data compression (coding)	Image synthesis	Image enhancement and restoration	Image analysis
Measuring the size of an object				
Visualizing the image of an object				
Reducing noise				
Compressing the binary flow				
Determining the color of an object				
Enhancing image contrast				
Recognizing a pattern in an image				

2 – This diagram represents very briefly a digital image or video compression application:

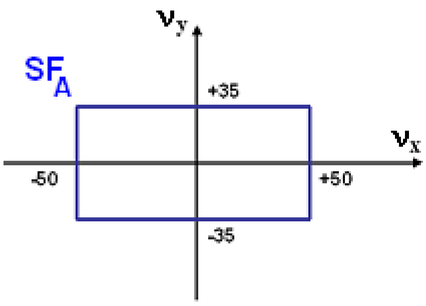
What is the object of each of these two main functions (boxes (1) and (2))?

3 – For an image analysis application, give two examples of data output from the Analysis block.



4 – <u>Sampling</u>

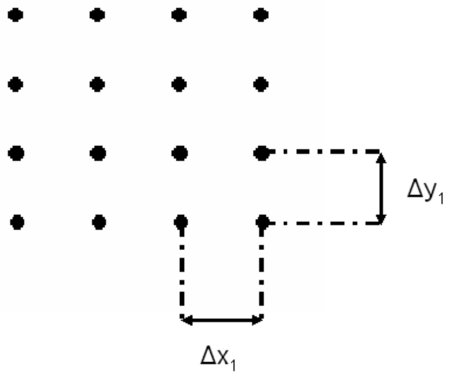
Let I_A be an analog input image (image signal $f_A(x, y)$). Its spectrum F_A is band-limited and the spectrum support $SF_A(\nu_x, \nu_y)$ is represented below:



- ν_x, ν_y : respectively horizontal and vertical spatial frequencies.

We sample this image I_A by a square sampling structure E_1 with three step values $(\Delta x_1, \Delta y_1)$:

- a) $\Delta x_1 = \Delta y_1 = 1/40$
- b) $\Delta x_1 = \Delta y_1 = 1/80$
- c) $\Delta x_1 = \Delta y_1 = 1/120$

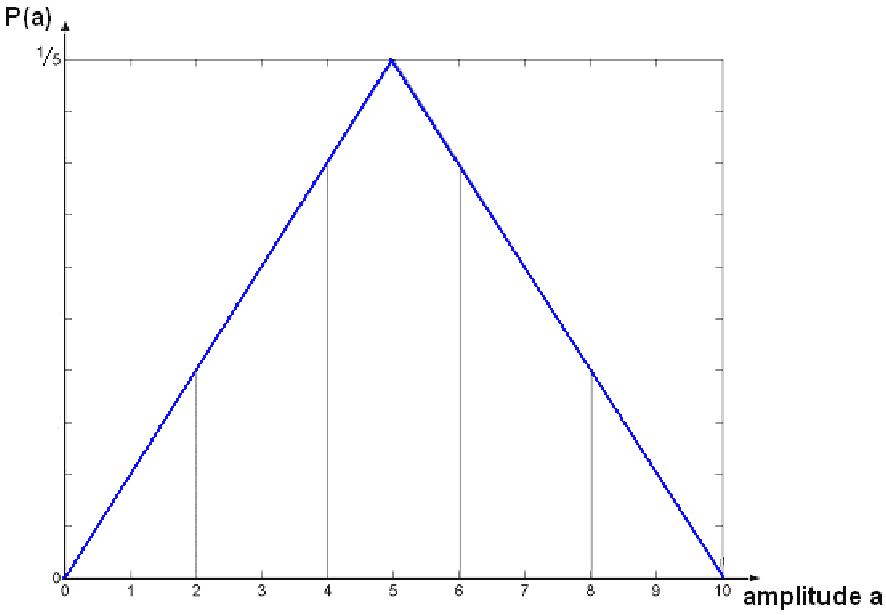


For each of these cases, a), b), and c), indicate if there is an *aliasing* effect in the horizontal structure or in the vertical structure. Justify your answer each time.

What conclusion do you reach about the differences with sampling purely time signals?

5 – Quantization

Given a sampled monochrome image, whose luminance follows the probability distribution below:



This is a symmetric linear law in relation to 5.
We want to quantize the luminance over 5 levels. To do this we fix beforehand 4 decision thresholds t_i so that the 5 intervals $[t_i, t_{i+1}[$ measure the same. So we have: $t_0=0$; $t_1=2$; $t_2=4$; $t_3=6$; $t_4=8$; $t_5=10$.

a) For each of these 5 intervals $[t_i, t_{i+1}[$; $i = \{0, 1, 2 , 3, 4, 5\}$ determine the optimal quantization level r_i that minimizes the mean square error.

b) From these 5 optimal quantization values r_1, r_2, r_3, r_4, r_5, what would be the optimal values for the four decision thresholds t_1, t_2, t_3, t_4 ?

We can note that we could simply reiterate this set of 2 stages to arrive at a law of optimal quantization, after stabilization of the quantization and decision values.